

$$(456)_8 \rightarrow (100101110)_2$$

$$(45.62)_8 \rightarrow (100101.11001)$$

$$(18.2A)_{16} \rightarrow (00011000.00101010)_2$$

$$11011000110 \rightarrow 1B86_H$$

$$456_8 \rightarrow \overset{1}{100} \overset{1}{101110} \rightarrow 12D \quad \text{OCTAL} \rightarrow \text{BIN} \rightarrow \text{HEX}$$

$$ABC_{16} \rightarrow \overset{1}{101010111100} \rightarrow 5274$$

$$456.25_8 \rightarrow 100101110.010101 \rightarrow (100.56)_2$$

OPERATION ON NUMBER SYSTEM

ADDITION

Addend

Augend

sum

SUBTRACTION

Substend

Minuend

difference

MULTIPLICATION

Multiplicand

Multiplier

Product

DIVISION

Dividend

Divisor

Quotient

R- Radix

R's complement $R^1s = R^n - N$

(R-1)'s complement.

ie 2 types of complement for any number system

45.

$$10^2 - 45 = 100 - 45 = 55$$

10's complement of 45 = 55

If there is n number of digits in integer part and m in ^{fractional} ~~decimal~~ part

$$2's \text{ comp of } (100)_2 = 2^3 - 100 = 1000 - 100 = 100 \quad (5ur)$$

$$(45)_8 \rightarrow 8^2 - 45 = 64 - 45$$

$$= 100 - 45 = (33)_8$$

$$(45)_{16} \rightarrow 16^2 - 45$$

$$256 - 45$$

$$100 - 45 = BB$$

$$\begin{array}{r} 8180 \\ 1 \end{array}$$

$$+6 \quad 16$$

$$\begin{array}{r} 16 \quad 254 \\ 16 \end{array}$$

$$\begin{array}{r} 7 \\ 4 \quad 8 \\ 100 - \\ 45 \end{array}$$

$$33$$

$$\begin{array}{r} 16 \\ 100 - \\ 45 \end{array}$$

$$11 \quad 11$$

$$(R-1)'s \text{ complement} \rightarrow R^n - N - R^{-m}$$

$$(45)_{10} - 9's \text{ comp} \rightarrow 10^2 - 45 - 10^0$$

$$\rightarrow 100 - 45 - 1 = 54$$

$$(45.45)_{10} - 100 - 45 - 10^{-2} = 55.54$$

$$(100)_2 = 2^3 - 100 - 2^{-6}$$

$$= 1000 - 100 - 1 = (011)_2$$

$$(45)_{16} = \begin{matrix} BB & BA \\ 16's \text{ comp} & 15^{th} \text{ comp} \end{matrix}$$

1 ABC 2 . 24 AB

D	9	45	55	-	54	Subtract every digit from 9-1
B	1	100	-	100	-	

D	7	45	-	33	-	32
---	---	----	---	----	---	----

H	15	45		BB		BA
---	----	----	--	----	--	----

Subst from 15

$$(1A \cdot BC2 \cdot 24AB) = ES4 \ 3D \cdot DB5A$$

$$= E543 \ D \cdot DB5A$$

FOR BINARY

100	011	1's comp
<hr/>		
100		2's comp

- SUBTRACTION.

If there is carry, cut that out and it will be right answer and if not take the complement and give negative.

eg:
$$\begin{array}{r} 1000 \\ 0100 \\ \times 100 \\ \hline \end{array}$$

Take 2's complement,

$$\begin{array}{r} 0100 \\ 1011 + \\ \hline 1100 \end{array}$$

\Rightarrow

	1000	+
	1100	
	<hr/>	
x	10100	$\Rightarrow 4$
		[Ignore carry]

SUBTRACTION WITH $(r-1)$'s complement

The subtraction of 2 positive nos M and N both of the same radix r is

1. Add the minuend with the $(r-1)$'s complement of the subtrahend
2. (a) If an end carry occur add 1 to the LSB (end around carry)
(b) If there is no carry occur take the $(r-1)$'s complement of the output and put a negative sign at the front.

eg: $1000 - 1000$ 8-4

$$\begin{array}{r} 1000 \\ \underline{100} \\ 1000 \\ \downarrow \\ 0100 \\ \underline{1011} \\ 1100 \end{array} \Rightarrow \begin{array}{r} 1000 \\ \underline{1100} \\ \times 0100 \end{array}$$

SUBTRACTION USING r 's COMPLEMENT

1. Add minuend with r 's complement of subtrahend
2. (a) for an end carry discard it
(b) if there is no carry take the r 's complement of o/p and place -ve. at front

2nds comp

$$\begin{array}{r} 1100 \\ \underline{1100} \\ \hline 1100 \\ 0011 \\ \hline 0100 \\ \rightarrow 1100 \\ \underline{0100} \\ \times 0000 \end{array}$$

One arithmetic zero

1's comp

(there is negative zero)

$$\begin{array}{r} 1100 \\ \underline{1100} \\ \hline 1100 \\ 0011 \\ \hline 1111 \\ - 0000 \\ \hline \text{[negative zero.]} \end{array}$$

Two Arithmetic zeros

BINARY CODES

n bits → Maxm of 2^n quantities
 2^n quantities → Minm of n bits.

BINARY CODED
 DECIMAL
 ↓
 diff from
 bin. codes.

DECIMAL CODES

	(BCD)	Excess 3	84-2-1	2421	Biquinary code
0	0000	0011	0000	0000	0100001
1	0001	0100	0111	0001	0000011
2	0010	0001	0110	0010	0000101
3	0011	0110	0101	0011	0001001
4	0100	0111	0100	0100	0010001
5	0101	1000	1011	1011	1000001
6	0110	1001	1010	1100	1000010
7	0111	1010	1001	1101	1000100
8	1000	1011	1000	1110	1001000
9	1001	1100	1111	1111	1010000

ERROR CHECKING:

Never possible: codes like (1010, 1011, 1100 etc.)
 In BCD

Biquinary code - Self error detecting code.
 These are weighted codes except excess 3.

Binary conversion ¹² 1100
 BCD 0001 0010

Biquinary code

0100001
 0100010
 0100100
 0101000
 0110000
 1000001
 1000010
 1000100
 1001000
 1010000

Nos are represented in computers either in binary or in decimal using a binary code

9's complement of a decimal

395. Sub from 9

0011 + 10

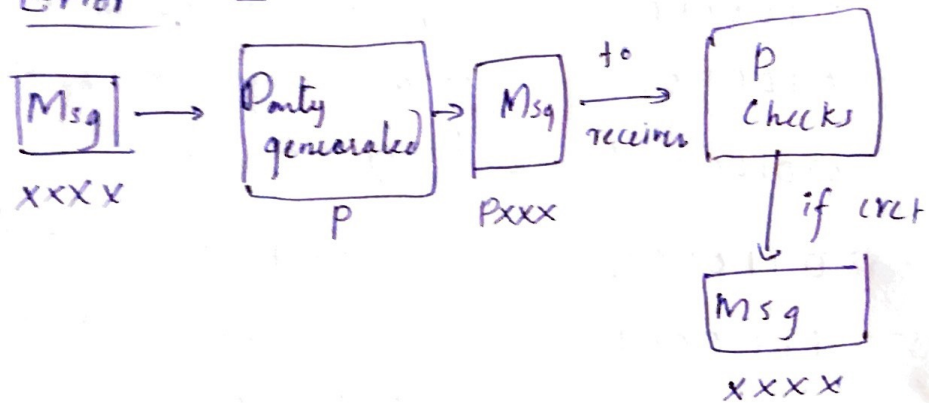
 604

→ For the first 1, 1 will be present at 2nd location and for the last at 1st locat place

One 1 present in 2nd place, other 1 in any other place other than first place, (0-3)

→ One 1 present in 1st place, any other 1 in any other place other than 2nd place. (5-9)

ERROR DETECTION CODES



Parity $\begin{cases} \text{Odd} \\ \text{Even} \end{cases}$

EVEN/ODD PARITY	Message	Correct/Not
1	1010	✓
1	1011	x
1	1100	✓

FREQUENCY OF ERROR

P P(odd)
1 0000
0 0001
...
1 1001

P P(even)
0 0000
1 0001
...
0 1001

To make parity odd
add 1 if it is odd
then 0

parity even →

REFLECTED CODE / GRAY CODE (Not predefined)

0 0000
1 0001
2 0011
3 0010
4 0110
5 0111
6 0101
7 0100

One bit and another differ by a single bit

Depends on starting value

- 8 1100
- 9 1101
- 10 1111
- 11 1110
- 12 1010
- 13 1011
- 14 1001
- 15 1000

ALPHANUMERIC CODES

6 bit internal code

8 bit EBCDIC Code

(Extended Binary Coded Decimal Interchange Code)

7 bit ASCII Code

(American Standard Code for Information Exchange)

BINARY LOGIC

Deals with variables that take on two

distinct values and with operations that get the meaning True/False

→ Consisting of binary variables and logical operations.

LOGIC GATES:

AND } Min 2 input

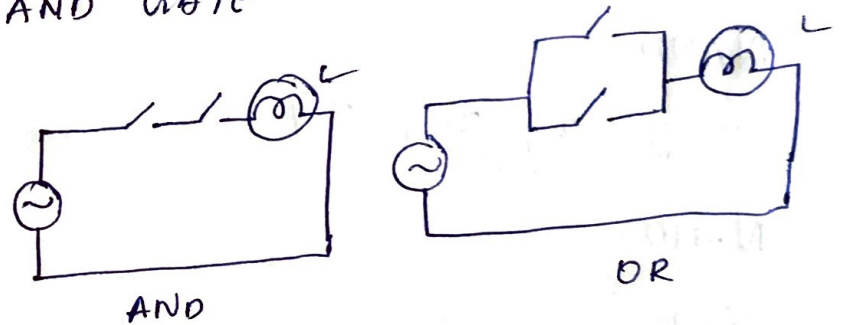
OR } Min 2 input

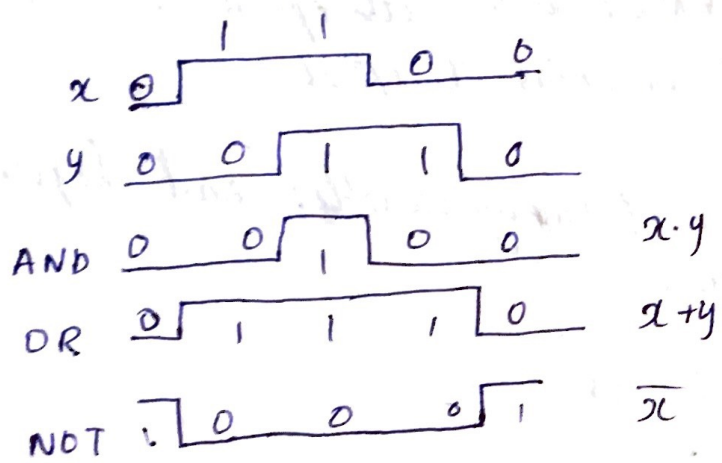
NOT → single i/p - INVERTER

Truth table: All combination of i/p → o/p

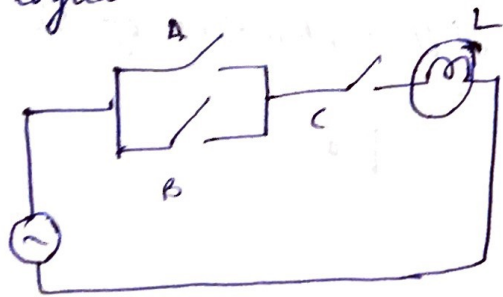


→ AND GATE





→ Express the following switchy circuit in logical notation.



$$L = (A + B) \cdot C$$

NAND

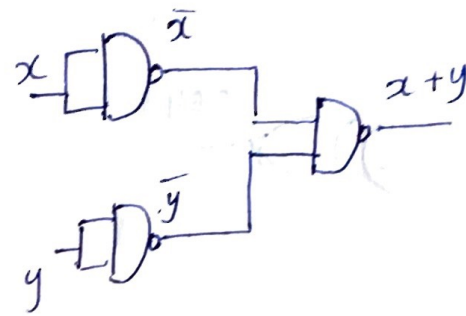
x	y	$x \cdot y$	$x + y$	$x \cdot \bar{y}$
---	---	-------------	---------	-------------------

NAND

→ AND



OR

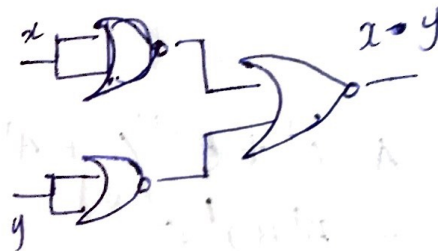


NOT



NOR

= AND



OR

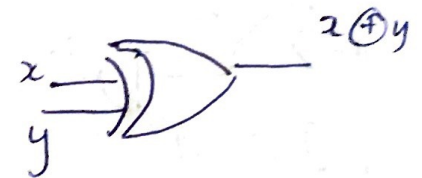


NOT



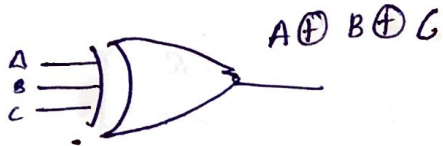
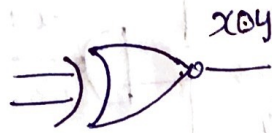
EXCLUSIVE OR GATE

x	y	
0	0	0
0	1	1
1	0	1
1	1	0



Exclusive Nor gate

x	y	$x \odot y$
0	0	1
1	0	0
0	1	0
1	1	1



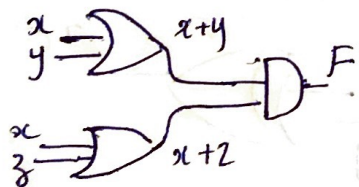
$$A \oplus B = AB' + A'B$$

$$(A \oplus B) \oplus C = AB'C' + (A \oplus B)'C + A'B'C + (A \oplus B)'C$$

$$A \odot B = A'B' + AB$$

Implement:

$$F = (x+y)(x+z)$$



BOOLEAN ALGEBRA

- may be defined with a set of elements
a set of operation and a no. of axioms/
postulates

set, \in , \notin , $\{ \}$

- A binary operator defined in a set S
of elements is a rule that assigns to
each pair of elements from S an
unique element from S

1. Closure $a * b = c$ $a, b, c \in S$
set of natural numbers + addition.
subtraction is not closed (-ve nos)

2. ASSOCIATIVE

$$(x * y) * z = x * (y * z) \text{ for all } x, y, z \in S$$

3. Commutative

$$x * y = y * x \text{ for all } x, y \in S$$

4. Identity element

$$e * x = x * e = x \text{ for every } x \in S$$

5. Inverse \rightarrow A set S having identity element w.r.t a binary operator is said to have an inverse whenever, for every element $x \in S$, there exists an element such that, $x * y = e$

e.g. Set of integers w.r.t addition, the integer inverse of a is,

$$a + (-a) = I = 0$$

6. Distributive law,

$$x * (y + z) = (x * y) + (x * z)$$

Multiplication and addition

Algebraic structures

\downarrow

Field - is a set of elements together with two binary operators each having the properties 1 to 5 and both operators combined to give property 6.

Field of real nos (set of real nos with two binary operations $+$ and \cdot)

Binary operator $+$ defines addition.

The additive identity is 0 .

The additive inverse defines subtraction.

Binary operator \cdot defines multiplication.

The multiplication identity is 1 .

The multiplication inverse defines division.

The only distributive law is \cdot over $+$

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

Boolean algebra

is an algebraic structure defined on a set of elements B together with two binary operators $+$ and \cdot , provided the following Huntington's postulates are satisfied

1. a) closure w.r.t the operator $+$

b) closure w.r.t the operator \cdot

2. a) An identity element w.r.t $+$,

designated by 0

$$x+0 = 0+x = x$$

(b) An identity element wrt \cdot , designated by 1,

$$x \cdot 1 = 1 \cdot x = x$$

(c)

3. a) Commutative wrt $+$; $x+y = y+x$

b) Commutative wrt \cdot ; $x \cdot y = y \cdot x$

4. a) \cdot is distributive over $+$:

$$x(y+z) = (x \cdot y) + (x \cdot z)$$

b) $+$ is distributive over \cdot :

$$x+(y \cdot z) = (x+y) \cdot (x+z)$$

5. For every element $x \in B$, there exists an element $x' \in B$ (complement of x) such that (a) $x+x' = 1$ and

$$(b) x \cdot x' = 0$$

6. There exists at least 2 elements

$x, y \in B$ such that $x \neq y$

Comparison:

1. B.A doesn't have associative law
2. The distributive property of $+$ over \cdot , is valid

3. B.A does not have an addition/multiplication inverse. Therefore there is no subtraction/division

4. Operator complement is not available in ordinary algebra

5. ~~boolean algebra~~ Ordinary algebra deals with real nos

6. B.A depends with only 2 nos 0 & 1

B.A should satisfy:

1. The elements of the set B

2. The rules of operation for the two binary operation.

3. That set of elements B, together with

the two operations satisfies the 6 Huntington's postulates

1. Closure
2. Identity
3. Commutative
4. Distributive
5. Complement
6. 2 distinct elements 0 & 1 and $0 \neq 1$

BASIC THEOREMS & PROPERTIES OF BOOLEAN ALGEBRA

1. DUALITY PRINCIPLE

One part can be obtained

The dual of an expression can be obtained by interchanging the operator & identity elements [on Huntington's postulate]

BASIC THEOREM

$$(a) x + 0 = x \quad (b) x \cdot 1 = x$$

$$(i) (a) x + x' = 1 \quad (b) x \cdot x' = 0$$

$$(ii) (a) x + x = x \quad (b) x \cdot x = x$$

$$(iii) (a) x + 1 = 1 \quad (b) x \cdot 0 = 0$$

$$(iv) \text{INVOLUTION THEOREM on } (x')' = x$$

$$\text{comm. (v) (a) } x + y = y + x \quad (b) x \cdot y = y \cdot x$$

$$\text{Assoc. (vi) (a) } x + (y + z) = (x + y) + z \quad (b) x \cdot (yz) = (xy) \cdot z$$

(viii) Distributive theorems

$$(a) x \cdot (y + z) = xy + xz$$

$$(b) x + (yz) = (x + y) \cdot (x + z)$$

$$(ix) a) (x + y)' = x' \cdot y' \quad [\text{DE MORGAN'S THEOREM}]$$

$$b) (x \cdot y)' = x' + y'$$

$$(x) (a) x + xy = x \quad [\text{ABSORPTION THEOREM}]$$

$$(b) x \cdot (x + y) = x$$

• Prove $x + x = x$

$$\Rightarrow (x + x) \cdot 1$$

$$\Rightarrow (x + x) \cdot (x + x')$$

$$\Rightarrow x + x\bar{x}$$

$$\Rightarrow x \quad [x\bar{x} = 0]$$

Q. $x \cdot x = x$

$$\Rightarrow (x \cdot x) + 1$$

$$\Rightarrow (x+1) \cdot x + (x \cdot x) + (x + \bar{x})$$

\Rightarrow

Proof:

\Rightarrow By basic theorem

\Rightarrow By truth table

\Rightarrow By Venn diagram

Boolean Function

$$F = x + y$$

A binary variable can have a value of 0/1
 A boolean function is an expression formed with Binary variables, the two binary operators OR and AND, the unary operator NOT, parenthesis and equal sign.

eg. $'xyz'$

$$F_1 = xyz'$$

$$F_2 = x'yz$$

$$F_3 = x'y'z + x'yz + xy'$$

$$F_4 = xy' + x'z$$

x	y	z	F ₁	F ₂	F ₃	F ₄
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	0	0	0	0
0	1	1	0	0	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0

$$x'y'z + x'yz + xy' = x'z(y' + y) + xy'$$

$$= x'z + xy'$$

Minimized function

- \rightarrow fewer no. of gates
- \rightarrow fewer no. of i/p's

Algebraic Manipulation

Literal \rightarrow a pre med / unpremed variable
 [complement]

$$(A+B+C)' = A' \cdot B' \cdot C'$$

GENERALIZED FORM OF DEMORGAN'S THEOREM

- Complement of a fn is obtained by interchanging AND and OR operators and complementing each literal

$$\begin{aligned} \text{eg. } F_1 &= x'y'z' + x'y'z \\ &= x'(y'z' + y'z) \\ &= x'((y+z)') \cdot (y'+z) \end{aligned}$$

Take dual and complement each literal

$$\Rightarrow (x+y+z) \cdot (x'+y'+z')$$

$$\begin{aligned} F_1 &= x'y'z' + x'y'z & F_2 &= x(y'z' + yz) \\ &= (x'+y+z) \cdot (x'+y'+z) & &= x + (y'+z') \cdot (y+z) \end{aligned}$$

Simplify the full bool. form to a min. no. of literals

$$\begin{aligned} [a] \quad xy + xy' \\ x(y+y') &= x \end{aligned}$$

$$\begin{aligned} [b] \quad (x+y) \cdot (x+y') \\ &= x \end{aligned}$$

$$\begin{aligned} [c] \quad xy'z + x'y + xy'z' \\ xy'[z+z'] + x'y \\ &= xy' + x'y \\ &= y(x+x') \\ &= y \end{aligned}$$

$$\begin{aligned} [d] \quad xz + x'y'z \\ z[x+x'y'] \\ &= z[x + x'y] \\ &= \underline{xyz} \\ &= z[(x+x') \cdot (x+y)] \\ &= [x+y]z \end{aligned}$$

$$[e] \quad A+B'$$

$$\begin{aligned} [e] \quad (A+B)' \cdot (A'+B')' \\ &= (A' \cdot B) \cdot (A \cdot B') \\ &= 0 \end{aligned}$$

$$\begin{aligned} [f] \quad y[wz' + wz] \\ \quad \quad \quad + xy \\ y[wz' + wz + x] \\ &= y[w+x] \end{aligned}$$

- Find the complement of the Boolean function and reduce them to a min. no. of literals.

$$a) (BC' + A'D)(AB' + CD')$$

$$\begin{aligned} \cancel{AB'BC'} + BC' &= AB'BC' + BC'CD' + A'DAB' + A'DCD' \\ &= D' = 1 \end{aligned}$$

$$b) B'D + A'BC' + ACD + A'BC$$

$$= D(A' + B') = \cancel{ACD} + B'D$$

$$= \cancel{D(A'C' + B)} = A'C'D' + B'D$$

$$c) \cancel{B'D} + A'BC'$$

$$= B'D'$$

$$= B'D + ACD + A'BC(C + C')$$

$$= \cancel{B'D} + ACD' + A'B$$

$$= (B + D')(A + B')(A' + C' + D')$$

$$= (AB + AD' + B'D')(A' + C' + D')$$

$$= ABC' + ABD' + AD'C' + B'D'A'$$

$$+ B'D'C'$$

=

$$c) - [(AB)'A] [(AB)'B]$$

$$(AB)' = 1$$

$$d) AB' + C'D'$$

$$(A' + B)(C' + D)$$

Obtain the truth table of the following.

$$\rightarrow F = xy + xy' + y'z$$

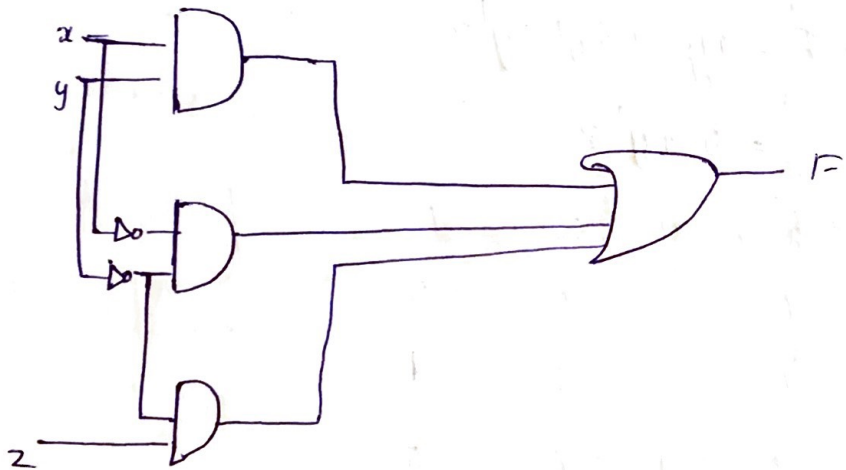
$$= x + y'z$$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

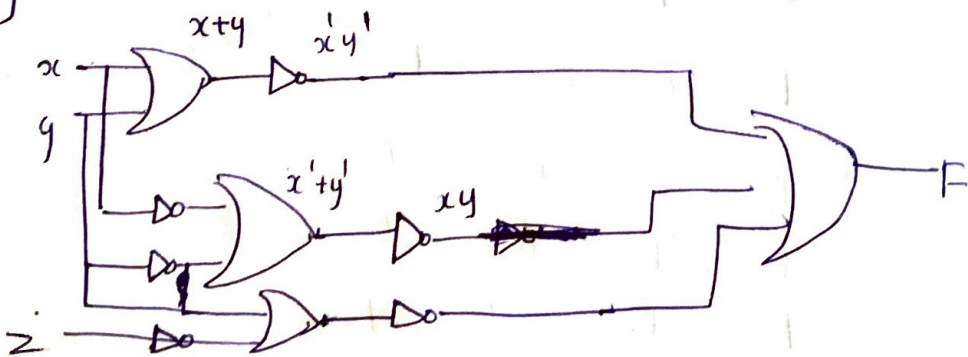
• $F = xy + x'y' + y'z$ $(x+y) \cdot [x+y]$ $[y+z]$

- a) Implement with AND OR & NOT gates
- b) Implement it with only OR & NOT gates
- c) Implement it with only AND & NOT gates

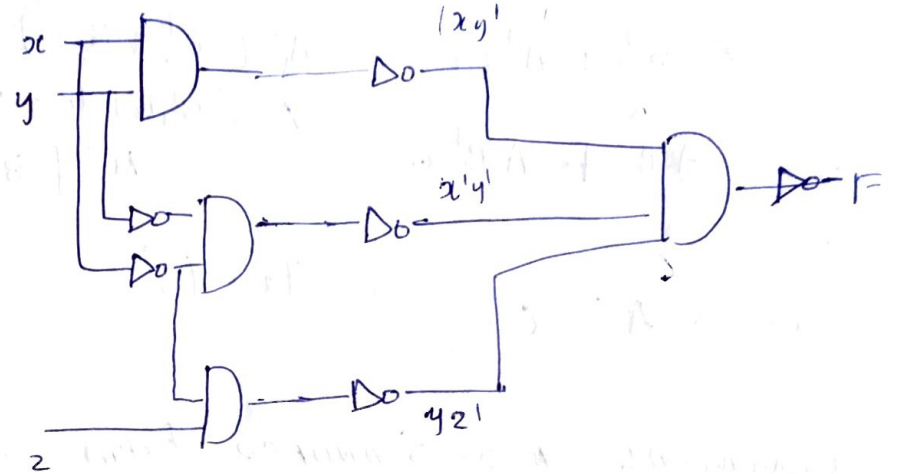
a)



b)



c)



→ Simplify the fns T_1 & T_2 to min no. of literals

A	B	C	T_1	T_2
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

$T_1 = A'B'C' + A'B'C + A'BC'$

$$A'B'(C+C') + A'BC'$$

$$= A'B' + A'BC' = A'[B' + BC']$$

$$= A'[(B'+B)(B'+C')] = A'[B'+C']$$

~~$$T_2 = AB + A'B'C$$~~

$$T_2 = T_1'$$

$$T_2 = A + BC$$

CANONICAL AND STANDARD FORM

CANONICAL $\left\{ \begin{array}{l} \text{Minterm} \rightarrow \text{standard product} \\ \text{Maxterm} \rightarrow \text{standard sum} \end{array} \right.$

In a canonical form all the variables should be there.

Standard form

eg $A + BC$

[sum of products]

Non standard form

$$A'(B+C')$$

(cannot be saved like that)

			Min term		Max term	
x	y	z	TERM	DESIGNATION	TERM	DESIGNATION
0	0	0	$x'y'z'$	m_0	$x+y+z'$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z$	M_1
0	1	0	$x'yz'$	m_2	$x'+y'+z'$	M_2
0	1	1	$x'yz$	m_3	$x'+y+z$	M_3
1	0	0	$xy'z'$	m_4	$x+y'+z'$	M_4
1	0	1	$xy'z$	m_5	$x+y'+z$	M_5
1	1	0	xyz'	m_6	$x+y+z'$	M_6
1	1	1	xyz	m_7	$x+y+z$	M_7

$$(m_0)' = M_0$$

x	y	z	F ₁	F ₂
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$F_1 = x'y'z + xy'z' + xyz$$

$$\Rightarrow m_1 + m_4 + m_7$$

$$F_1(x,y,z) \Rightarrow \sum (1, 4, 7)$$

$$F_2 = m_3 + m_5 + m_6 + m_7$$

$$F_2(x,y,z) = \sum (3, 5, 6, 7)$$

$$(F_1)' = x'y'z' + x'yz' + x'yz + xy'z + xyz'$$

$$(F_1)'' = F_1$$

$$= (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z)$$

$$= M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$$

$$= \Pi (0, 2, 3, 5, 6)$$

$$F_2' = \sum (3, 5, 6, 7)$$

$$F_2 = \Pi (0, 1, 2, 4)$$

→ Boolean form expressed as a sum of minterm or product of maxterm are said to be in canonical form

SUM OF MINTERMS:

Express the boolean fun $F = A + B'C$ in a sum of minterm form

$$F = ABC + AB'C + ABC' + AB'C' + A'B'C$$

$$F = 1 = \sum (1, 4, 5, 6, 7)$$

$$A = A[B+B']$$

$$\Rightarrow AB + AB'$$

$$\Rightarrow AB(C+C') + AB'(C+C')$$

$$\Rightarrow ABC + ABC' + AB'C + AB'C'$$

$$B'C \Rightarrow (A+A')B'C$$

$$\Rightarrow AB'C + A'B'C$$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

- Express the boolean fn $F = xy + x'z$ as a product of maxterm form

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$\pi [0, 2, 4, 5]$$

Maxterm = 0

Minterm = 1

CONVERSION BETWEEN CANONICAL FORM

Complement of a function expressed as the sum of minterm equals the sum of minterm missing from original term

Eg: $F(A, B, C) = \sum(1, 4, 5, 6, 7)$

$$F'(A, B, C) = \sum(0, 2, 3) = m_0 + m_2 + m_3$$

$$F(A, B, C) = (m_0 + m_2 + m_3)'$$

$$= m_0' \cdot m_2' \cdot m_3'$$

$$= M_0 \cdot M_2 \cdot M_3$$

$$= \pi(0, 2, 3)$$

$$F(x, y, z) = \pi[0, 2, 4, 5]$$

⇓

$$F(x, y, z) = \sum(1, 3, 6, 7)$$

STANDARD FORMS $\left\{ \begin{array}{l} \text{sum of products} \\ \text{product of sums} \end{array} \right.$

$$F_1 = y' + xy + x'yz'$$

$$F_2 = x(y' + z)(x' + y + z' + w)$$

NON-STANDARD FORM :-

$$F_3 = (AB + CD)(A'B' + C'D')$$

$$\Downarrow$$

$$F_3 = ABC'D' + A'B'CD \quad (\text{standard form})$$

- Express following in sum of min terms and product of maxterm

$$\begin{aligned} \text{u) } F(A, B, C, D) &= D(A + B) + B'D \\ &= A'D + BD + B'D \end{aligned}$$

$$F = \sum (1, 3, 5, 7, 9, 11, 13, 15)$$

$$F = \prod (0, 2, 4, 6, 8, 10, 12, 14)$$

$$6) F(A, B, C, D) = (A+B'+C)(A+B')(A+C'+D) \\ (A'+B+C+D)(B+C'+D')$$

$$= (A+B')(C)$$

$$= \cancel{AA} + \cancel{AB'} + \cancel{AB'}$$

$$F = \sum (0, 1, 2, 8, 10, 12, 13, 14, 15)$$

$$\prod (3, 4, 5, 6, 7, 9, 11)$$

$$\bullet F(x, y, z) = 1$$

$$F(x, y, z) = \sum (0, 1, 2, 3, 4, 5, 6, 7)$$

No max term

$$xy = xy(z+z')$$

$$\bullet F(x, y, z) = (xy+z)(y+xz)$$

$$= \cancel{xyx} + \cancel{xyxz} + yz + xz$$

$$= xy + yz + xz$$

$$\sum (3, 5, 6, 7) \prod (0, 1, 2, 4)$$

SIMPLIFICATION OF BOOLEAN FUNCTION

- MAP METHOD - VEITCH - KARNATH

KARNAUTH MAP / K-MAP:

Adjacent box differ by 1 bit.

Total no. of boxes = 2^n (n-variable)

Make a combination power of 2 no. of boxes and produce the output

2 variable Kmap:

	x	\bar{x}	y
\bar{y}	m_0	m_1	
y	m_2	m_3	

$$F = xy + \bar{x}y$$

	x	\bar{x}	y
\bar{y}			1
y			1

$$F = y$$

	y	\bar{y}
x	$x\bar{y}$	$x\bar{y}$
\bar{x}	$\bar{x}y$	$\bar{x}y$

3 variable K-MAP

	yz	$\bar{y}z$	$y\bar{z}$	$\bar{y}\bar{z}$
x	m_0	m_1	m_3	m_2
\bar{x}	m_4	m_5	m_7	m_6

$\underbrace{\hspace{10em}}_2$

$$F = x'yz' + \bar{x}y'z' + xy'z'$$

	$\bar{y}z$	$\bar{y}\bar{z}$	yz	$y\bar{z}$
\bar{x}			1	1
x	1	1		

$$F = x\bar{y} + \bar{x}y$$

$$F = x'yz + xy'z' + xyz + xy'z'$$

	$y'z'$	$y'z$	yz	yz'
x'		1		
x	1		1	1

$$F = yz + x\bar{z}$$

$$F = x'y'z' + xy'z' + xy'z + xyz$$

1			1
1			1

$$F = y'z' + yz'$$

$$\Rightarrow F = A'C + A'B + AB'C + BC$$

	BC	BC	BC	BC'
A		1	1	1
\bar{A}		1	1	
A		1	1	

$$F = C + \bar{A}B$$

$$\Rightarrow F(x,y,z) = \sum(0,2,4,5,6)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
x'	1			1
x	1	1		1

$$F = \bar{z} + xy$$

\Rightarrow 4-Variable K-map

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$	
$\bar{A}\bar{B}$	m_0	m_1	m_3	m_2	$16 \rightarrow 1$ $8, 9 \rightarrow 1$ $4, 5, 9 \rightarrow 2$ $2 \rightarrow 3$ $1 \rightarrow 4$
$\bar{A}B$	m_4	m_5	m_7	m_6	
AB	m_{12}	m_{13}	m_{15}	m_{14}	
$A\bar{B}$	m_8	m_9	m_{11}	m_{10}	
	$\underbrace{\hspace{10em}}_B$ $\underbrace{\hspace{10em}}_C$				

eg: Simplify.

$$F(w,x,y,z) = \sum(0,1,2,4,5,6,8,9,12,13,14)$$

1	1		1
1	1		1
1	1		1
1	1		1

$$F = \bar{x} + \bar{y} + \bar{z} + \bar{w} + \bar{z} + w'z$$

2) $F = A'B'C' + B'CD' + ABCD' + AB'C$

1	1		1
			1
1	1		1

$$F = B'b' + B'C' + A'CD'$$

FIVE VARIABLES K-MAP

AB \ CDE	000	001	010	011	110	111	101	100
00	0	1	2	3	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

Add MSB

00 01 11 10 00 11 01 00

Add MSB 0

Add MSB 1

000 001 011 010 110 111 101 100

Simplify (A, B, C, D, E) = {0, 2, 4, 8, 9, 11, 13, 15, 17, 21, 25, 27, 29}

AB \ CDE	$\bar{C}\bar{D}\bar{E}$	$\bar{C}\bar{D}E$	$\bar{C}D\bar{E}$	$\bar{C}DE$	$C\bar{D}\bar{E}$	$C\bar{D}E$	$CD\bar{E}$	CDE
$\bar{A}\bar{B}$	1			1	1			1
$\bar{A}B$		1	1			1	1	
AB		1	1			1	1	
$A\bar{B}$		1	1			1	1	

$$F = BE + A\bar{D}C + \bar{A}\bar{B}\bar{E}$$

SIX VARIABLE K-MAP

	DEF							
ABC	000	001	011	010	110	111	101	100
000	0	1	3	2	6	7	5	4
001	8	9	11	10	14	15	13	12
011	24	25	27	26	30	31	29	28
010	16	17	19	18	22	23	21	20
110	48	49	51	50	54	55	53	52
111	56	57	59	58	62	63	61	60
101	40	41	43	42	46	47	45	44
100	32	33	35	34	38	39	37	36

31 → 15, 23, 24, 30, 27, 63

64 → 1

32 → 1 variable

16 → 2 variables

8 → 3 variables

4 → 4 variables

2 → 5 variables

1 → 6 variables

Product of sum simplification

eg: Simplify the following boolean function

100 (a) SOP & (b) POS

$$F(A, B, C, D) = \sum (0, 1, 2, 5, 8, 9, 10)$$

(a) F =

	CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD
$\bar{A}\bar{B}$	1	1	0	1
$\bar{A}B$	0	1	0	1
$A\bar{B}$	0	0	0	0
AB	1	1	0	1

$$F = \bar{A}\bar{C}D + \bar{A}B\bar{D} + B\bar{C}$$

b) $F = (C+D)(\bar{A}+B)(\bar{B}+D)$

$$F' = \bar{A}B + CD + B\bar{D}$$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1

(a) F =

(b) F' =

1 1 1 0

$$f = \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}z + xy\bar{z}$$

$$= \bar{x}z(\bar{y}+y) + xz(\bar{y}+y)$$

$$= \bar{x}z + xz$$

(a) $F = x'z + z'$

(b) $F' = xz + z'$

$$f = (x' + z')(z + z')$$

$$f' = (\bar{x} + y + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z)(\bar{x} + y + \bar{z})$$

(3) $F = (A' + B' + C)(B + D)$

$$F' = ABC' + B'D$$

Mark 0's and mark 1 in all remain boxes.

		C			
		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$		0	1	1	0
$\bar{A}B$		1	1	1	1
AB		0	0	1	1
$A\bar{B}$		0	1	1	0

$$F = \bar{A}B + BC + \bar{B}D$$

$$F = \bar{B}\bar{D} + ABC$$

$$F = (B + D)(\bar{A} + \bar{B} + C)$$

→ Simplify

a) $F(x, y, z) = \sum (2, 3, 6, 7)$

b) $F(A, B, C, D) = \sum (7, 13, 14, 15)$

c) $F(A, B, C, D) = \sum (4, 6, 7, 15)$

1. Obtain the simplified expression in POS

(i) $F(x, y, z) = \pi(0, 1, 4, 5)$

(ii) $F(A, B, C) = \pi(0, 1, 2, 3; 4)$

(iii) $F(W, x, y, z) = \pi(1, 3, 5, 7, 15, 13)$

a) x, y, z

		1	1
		1	1

$$F(x, y, z) = \pi(4)$$

b)

			\overline{CD}
		1	0
1	1	1	1
		0	0

$$F = A\overline{B}C + AB\overline{D} + ABCD$$

$$= AB(C+D) + A\overline{B}C$$

c)

1		1	1
		1	

$$F = BCD + \overline{A}B\overline{D}$$

(i) $F = 1$

(ii) $F = (B + \overline{C}) (A + B) (A + C + D)$

(iii)

	0	0
0	0	0
0	0	0

$$F = (B + \overline{D}) (\overline{A} + \overline{B} + D)$$

$$= BD + \overline{A}\overline{B}D$$

$$= (\overline{B} + \overline{D}) (A + \overline{D})$$

DON'T CARE CONDITION

~~ABC~~ → 4 W

- Simplify the boolean form

$$F(w, x, y, z) = \sum (1, 3, 7, 11, 15)$$

$$d(w, x, y, z) = \sum (0, 2, 5)$$

	\overline{yz}	\overline{yz}	yz	yz
$\overline{w}x$	0	1	1	0
$\overline{w}\overline{x}$	0	0	1	0
$w\overline{x}$	0	0	1	0
wx	0	0	1	0

Combining 1's → $F(w, x, y, z) = yz + \overline{w}z$

OR $F = yz + \overline{w}z$

Combining 0's → $\overline{z} + w\overline{y}$

$$F = z(\overline{w} + y)$$

Don't care condition is used if it goes for a large combination.

2) $F = (y' + z'z')$

$$d = yz + zy$$

$$3) F = B'C'D' + BCD' + ABCD'$$

$$d = B'CD' + A'BC'D$$

2)

		yz			
		y \bar{z}	$\bar{y}z$	yz	$y\bar{z}$
\bar{x}	1	1	x	1	
x	1	1	x	x	

$$F = 1$$

3)

		CD			
		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	x	0	x	
$\bar{A}B$	0	x	0	1	
AB	0	0	0	1	
$A\bar{B}$	1	0	0	x	

$$F = C\bar{D} + A'B'D'$$

Simplify (a) in SOP and (b) in POS

a) $F = A'B'D' + A'CD + A'BC$

$$d = A'BC'D + ACD + AB'D$$

b) $F = W'(x'y + x'y' + xy'z) + x'z'(y + w)$

$$d = W'x(y'z + yz') + Wyz$$

(c) $F = ACE + A'CD'E' + A'C'DE$

$$d = DE' + A'B'E + AD'E'$$

a)

		CD			
		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	1	1	
$\bar{A}B$	0	x	1	1	
AB	0	0	x	0	
$A\bar{B}$	x	0	x	x	

$$F = \bar{A}C + \bar{B}\bar{D}$$

$$F' = A + C\bar{D} + B\bar{C}$$

$$F = \bar{A}(C + \bar{D})(\bar{B} + C)$$

b)

		yz			
		$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	1	1	1	1	
$\bar{w}x$	0	x	1	1	
wx	0	0	x	0	
$w\bar{x}$	1	0	x	1	

$$F = w'z + \bar{w}\bar{x} + yz + \bar{x}\bar{z}$$

$$F' = x\bar{y} + \bar{w}x + w'z$$

$$f = (\bar{x} + y)(\bar{w} + x)(w' + z)$$

$$= x(w' + z)(x' + z)$$

$$F = \bar{w}z + \bar{x}\bar{z}$$

$$F = (w' + z)(x' + z)$$

c)

		DE			
		$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{A}\bar{C}$	1	x	1	x	
$\bar{A}C$	1	x	0	x	
AC	x	1	1	x	
$A\bar{C}$	x	0	0	x	

$$F = C\bar{D} + AC + A'CE'$$

$$F' = A\bar{C} + C'E' + A'CE$$

$$= (A + C)(C + E)(A + C'E')$$

The following boolean expression $BE + B'DE$ is a simplified expression of version of expression

$$A'BCE + B'CD'E + BC'D'E + A'B'DE + B'CD'E$$

Are there any don't care conditions? If so

What are they?

AB	CDE							
	$\bar{C}\bar{D}\bar{E}$	$\bar{C}\bar{D}E$	$\bar{C}D\bar{E}$	$\bar{C}DE$	$C\bar{D}\bar{E}$	$C\bar{D}E$	$CD\bar{E}$	CDE
$\bar{A}\bar{B}$				1				1
$\bar{A}B$		1	1			1	1	
AB		1	*			*	1	
AB				1	*			*

$$F = \bar{A}BE + \bar{A}\bar{B}C\bar{D}\bar{E} + \bar{A}\bar{B}CDE$$

$$= \bar{A}B\bar{C}DE + \bar{A}BC\bar{D}\bar{E} + \bar{A}\bar{B}CDE$$

THE TABULATION METHOD / QUINE, McLOSKEY METHOD

→ Prime implicants

1. Finding the prime implicants

2. Determination of essential prime implicants

Simplify $F = \sum (0, 1, 2, 8, 10, 11, 14, 15)$

1. Determination of prime implicants.

(a)

W	x	y	z	
0	0	0	0	(0,1)
1	0	0	0	(0,2)
2	0	0	1	(0,8)
8	1	0	0	(2,10)
10	1	0	1	(8,10)
11	1	0	1	(10,11)
14	1	1	1	(10,14)
15	1	1	1	(11,15)

(Based on no. of 1's)

(c)

W	x	y	z	
-	0	-	0	(0,2,8,10)
-	0	-	0	(0,8,2,10)
1	-	1	-	(10,14,11,15)
1	-	1	-	(10,14,11,15)

$$F = w'x'y'z + x'z' + wxy$$

(a)

0
1
2
8
10
11
14
15

(b)

x0,1 (1)
 x0,2 (2) *
 0,8 (8) ✓
 (2,10) (8) ✓
 (8,10) (2) *
 10,11 (1) *
 10,14 (4) 0
 11,15 (4) 0
 14,15 (1) *

(c)

0,2,8,10 (2) 8
 0,8,2,10 (2) 8
 10,11,14,15 (1) 4
 10,14,11,15 (1) 4

000 | (0,1)
 000 |

$w'x'y' + x'z'$
 $+ wy$

0	0	0
0	0	0
0	0	0
1	0	0

(0,2,8,10)

1	0	1	0
1	0	1	0
1	1	1	0
1	1	1	1

(10,11,14,15)

SIMPLIFY $F(wxyz) = \sum (1,4,6,7,8,9,10,11,15)$

w x y z

1 0 0 0 1

4
8
8

(b)

6 x(1,9) (8)
 9 x(4,6) (12)
 10 (8,9) (1) ✓
 7 (2,10) (2) ✓
 11 x(6,7) (11)
 15 9,11 (2) ✓
 10,11 (1) ✓
 7,15 (8)
 11,15 (4)

0	1	1	1
1	0	1	1
1	0	1	1
1	1	1	1

0	0	0	1
0	0	0	1
0	1	0	0
0	1	1	0
0	1	1	1

(c)

8,9,10,11 (1,2)
 (8,10,9,11) (1,2)

0	1	1	1
1	1	1	1
0	0	0	1
1	0	0	1
0	1	0	0
0	1	1	0

1	0	0	0
1	0	1	1

Prime implicants

$x'y'z$, $w'xz'$, $w'xy$

xyz , wyz , wx'

PRIME IMPLICANT TABLE

	1	4	6	7	8	9	10	11	15
$x'y'z$ (1,9)	*					*			
$w'xz'$ (4,6)		*	*						
$xw'xy$ (6,7)			*	*					
xyz (7,15)				*					*
wyz (11,15)								*	*
wx' (8,9,10,11)					*	*	*	*	*
	✓	✓	✓	✓	✓	✓	✓	✓	✓

$$F = x'y'z + w'xz' + wx' + xyz$$

→ Simplify using tabulation

a) $F(ABC, DEF) = \sum (10, 28, 52, 60)$

(a)

(b)

<u>0010100</u>	001-100 (8)	0-1-100
0011100	0-1 0100 (32)	0-1-100
0110100	0-11100 (32)	
<u>0111100</u>	011-100 (8)	

a) $F(A, B, C, D, E, G) = \sum (20, 28, 38, 39, 52, 60, 102, 103, 127)$

⇒ $ABCDEF + A'CEF'G' + B'CDF$

DONT CARE CONDITION

↳ Included in the determination of prime implicants

↳ Not included in setting up the prime implicant table.

COMBINATIONAL CIRCUITS

DESIGN PROCEDURE

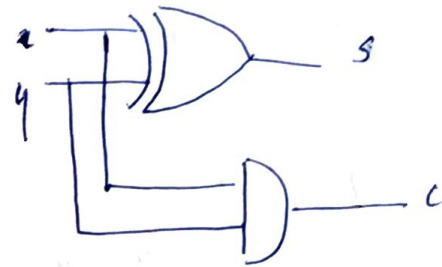
1. Problem statement
2. No of i/p and no. of o/p's
3. Give some symbols and name to i/p & o/p
4. Find relationship b/w i/p and o/p using a truth table
5. Derive expression for o/p variable w.r.t i/p variable
6. Simplify expression
7. Draw logic design.

ADDERS:

x	y	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$S = x'y + xy'$$

$$C = xy$$



FULL ADDER

x	y	z	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = x'y'z + x'y'z' + x'y'z + xy'z'$$

$$C = x'y'z + x'y'z' + xy'z' + xy'z$$

$$S = x \oplus y \oplus z$$

$$= (x'y + y'x) \oplus z = (x'y + y'x)'z + (x'y + y'x)z'$$

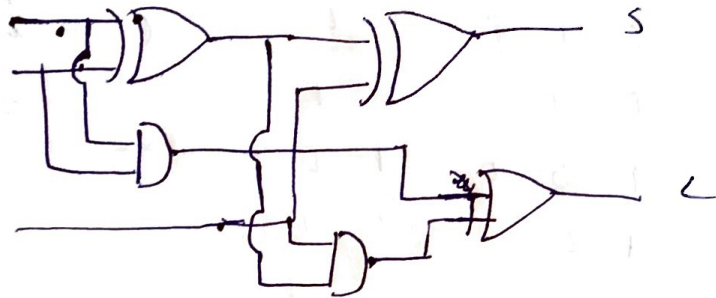
$$= x'y'z + x'y'z' + x'y'z' + x'y'z'$$

$$= (\bar{x} + y')(x + y)z + x'yz' + xy'z'$$

$$= x'yz + x'y'z + x'yz' + xy'z'$$

$$C = yz + xz + xy$$

	$\bar{y}z$	$y\bar{z}$	yz	$y\bar{z}$
\bar{x}			1	1
x	1	1	1	1



2 HA + 1 OR

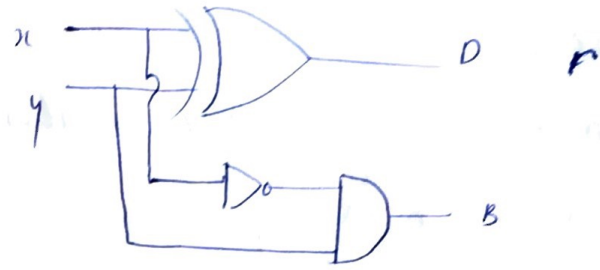
SUBTRACTOR:

→ HS

x	y	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$D = x'y + xy'$$

$$B = x'y$$



FULL SUBTRACTOR

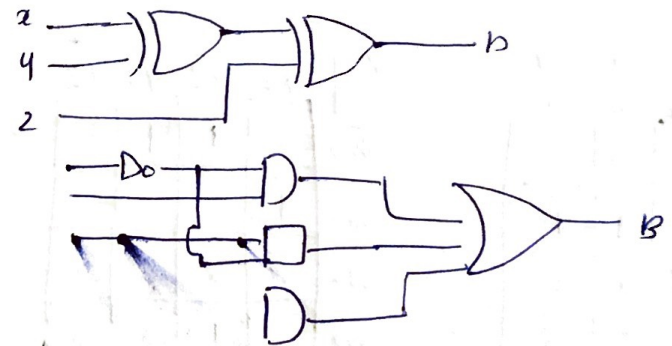
x	y	z	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$D =$

$$B = x'z + x'y + yz$$

$x'yz$

	1	1	1
		1	

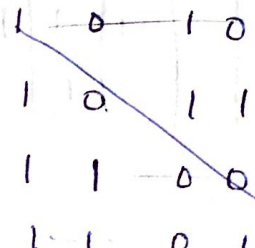


CODE CONVERTERS

- Design a BCD to excess 3 code converter.

BCD				Ex-3			
A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

$\bar{A}\bar{B}\bar{C}D$



W

		CD			
		$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	$\bar{A}B$				
$A\bar{B}$	AB	X	X	X	X
$\bar{A}\bar{B}$	$A\bar{B}$	1	1	X	X

Y

		YZ			
		$\bar{Y}\bar{Z}$	$\bar{Y}Z$	YZ	$Y\bar{Z}$
$\bar{W}\bar{X}$	$\bar{W}X$		1	1	1
$\bar{W}\bar{X}$	WX	1			
$W\bar{X}$	WX	X	X	X	X
$W\bar{X}$	WX		1	X	X

$$W = A + BD + BC$$

$$X = \bar{B}D + B\bar{C}\bar{D} + \bar{B}C$$

Y

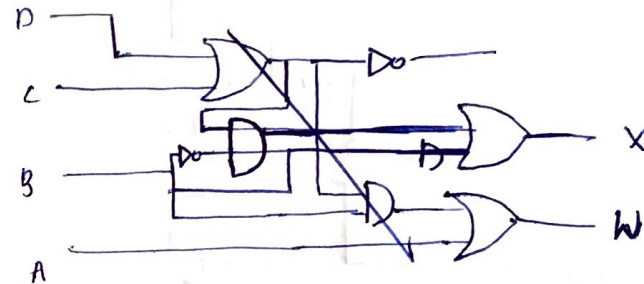
		CD			
		$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	$\bar{A}B$	1		1	
$A\bar{B}$	AB	X	X	X	X
$\bar{A}\bar{B}$	$A\bar{B}$	1		X	X

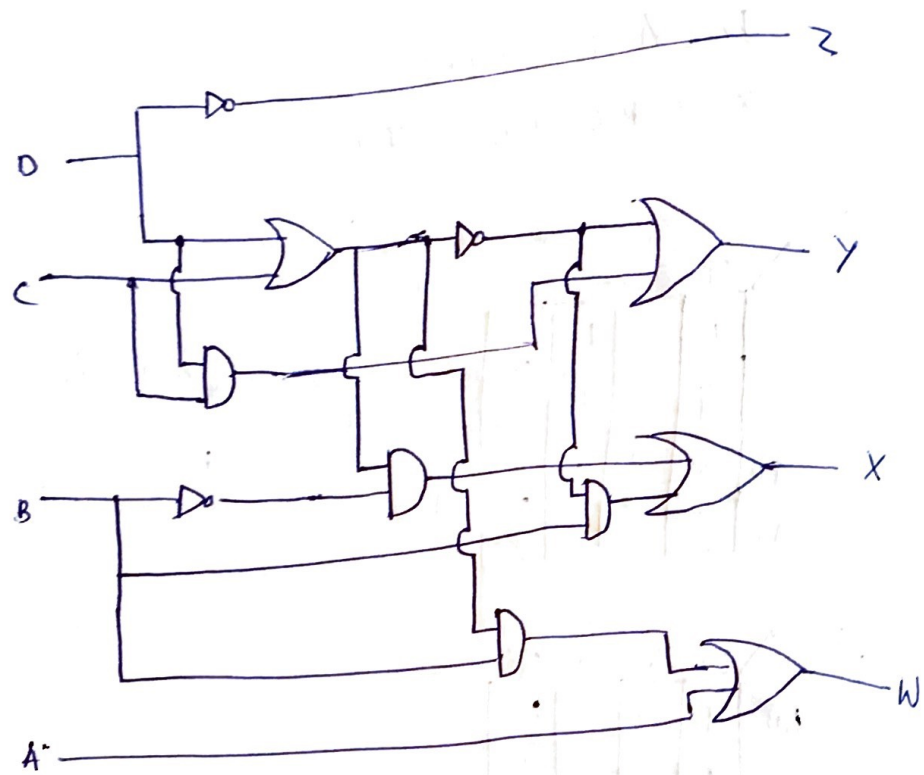
$$Y = CD + \bar{C}\bar{D}$$

$$Z = \bar{D}$$

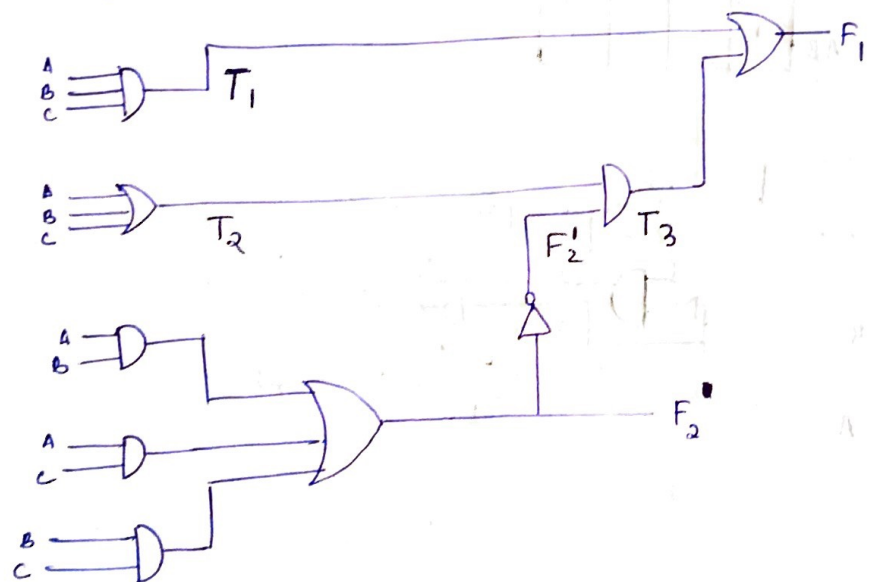
Z

		CD			
		$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	$\bar{A}B$	1			1
$A\bar{B}$	AB	X	X	X	X
$\bar{A}\bar{B}$	$A\bar{B}$	1		X	X





Analysis procedure



$F_2 = AB + AC + BC$ $T_1 = ABC$ $T_2 = A+B+C$

$T_3 = T_2 \cdot F_2'$

$T_3 = (A+B+C)(AB+AC+BC)'$
 $= (A+B+C)(\bar{A}+\bar{B})(\bar{A}+\bar{C})(\bar{B}+\bar{C})$

$F_1 = T_1 + T_3$
 $\Rightarrow ABC + T_3$

$= (AB + \bar{A}B + \bar{A}C + \bar{B}C)(\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{C})$
 $= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C$

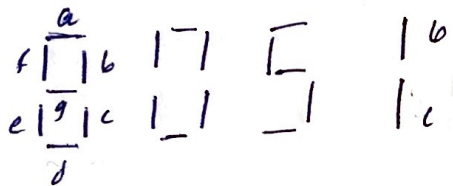
$F_1 = T_1 + T_3$

$= ABC + T_3$

$= ABC + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C$

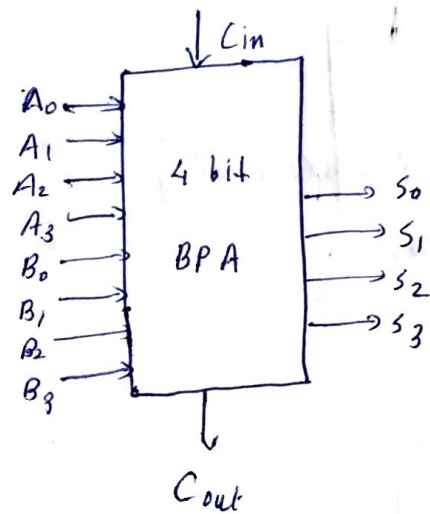
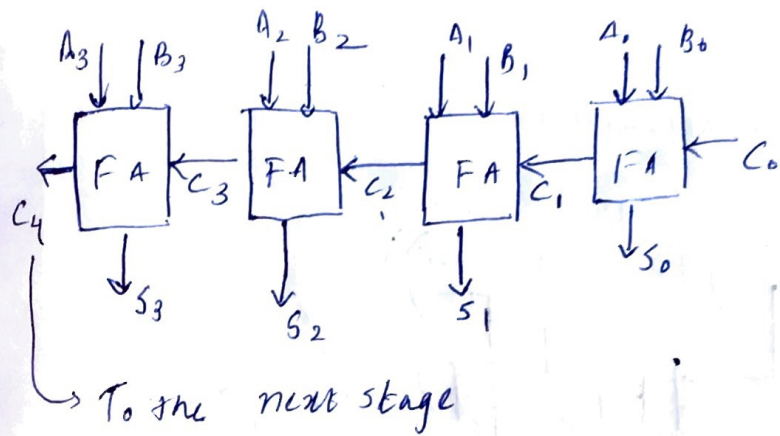
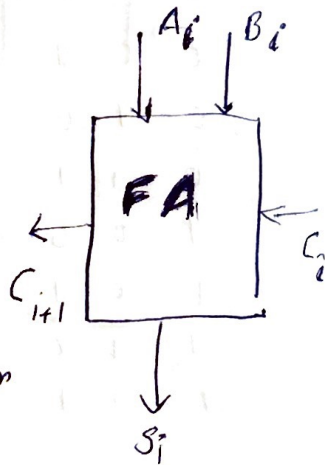
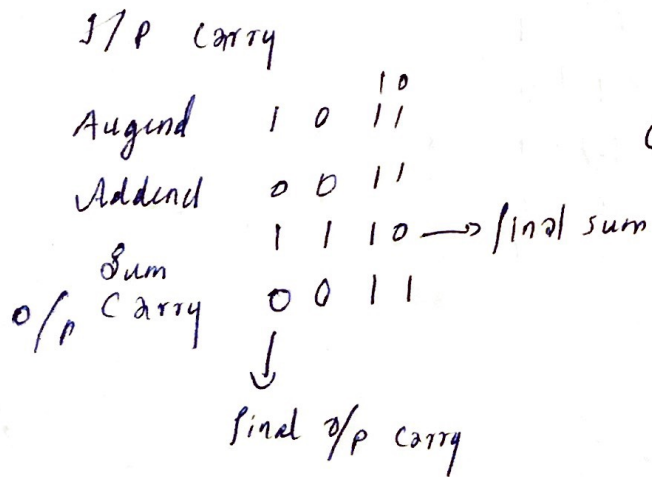
A	B	C	F ₁	F ₂
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

A BCD to seven segment decoder is a combinational circuit that accepts a decimal digit in BCD and generates the appropriate outputs for selection of segments in a display indicator used for displaying the decimal digits. Design a BCD to seven segment decoder circuit.

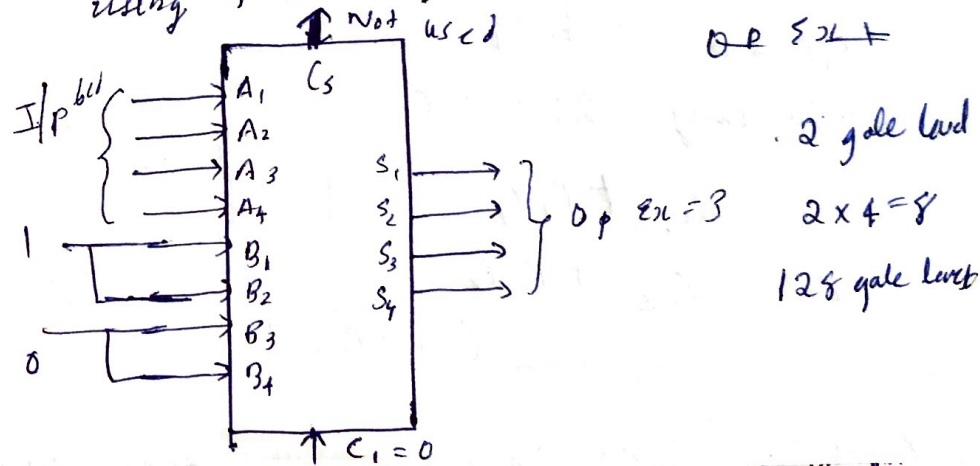


W	X	Y	Z	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0

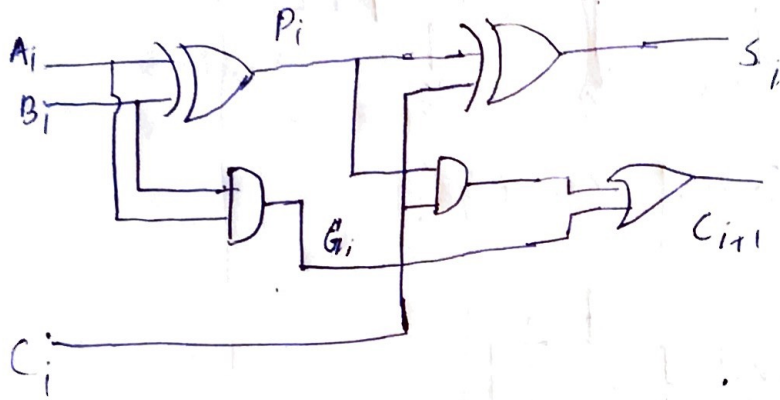
BINARY PARALLEL ADDER



Eq. Design a BCD to excess 3 code converter using 4 bit BPA?



CARRY PROPAGATION



Output carry - 2 gate levels.

Carry propagation =

Carry generation

Look ahead carry

$$P_i = A_i \oplus B_i$$

$$G_i = A_i \cdot B_i$$

$$\text{o/p sum} = P_i \oplus C_i$$

$$\text{o/p carry} = G_i + P_i C_i$$

$$C_2 = G_1 + P_1 C_1$$

$$C_3 = G_2 + P_2 C_2$$

$$= G_2 + P_2 (G_1 + P_1 C_1)$$

$$= G_2 + P_2 G_1 + P_2 P_1 C_1$$

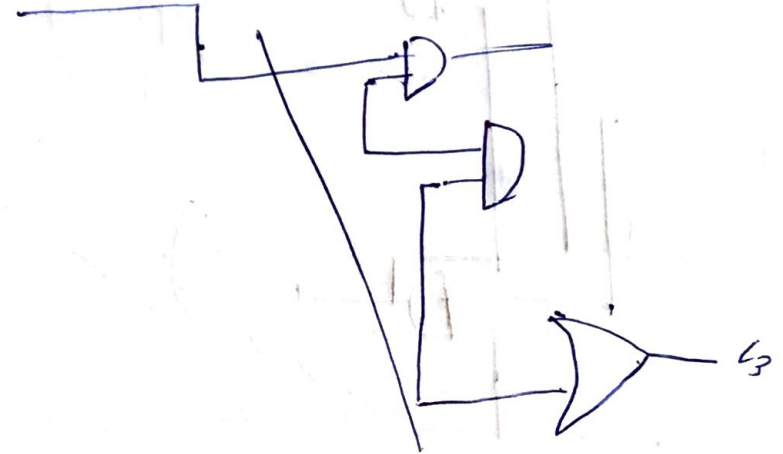
$$C_4 = G_3 + P_3 C_3$$

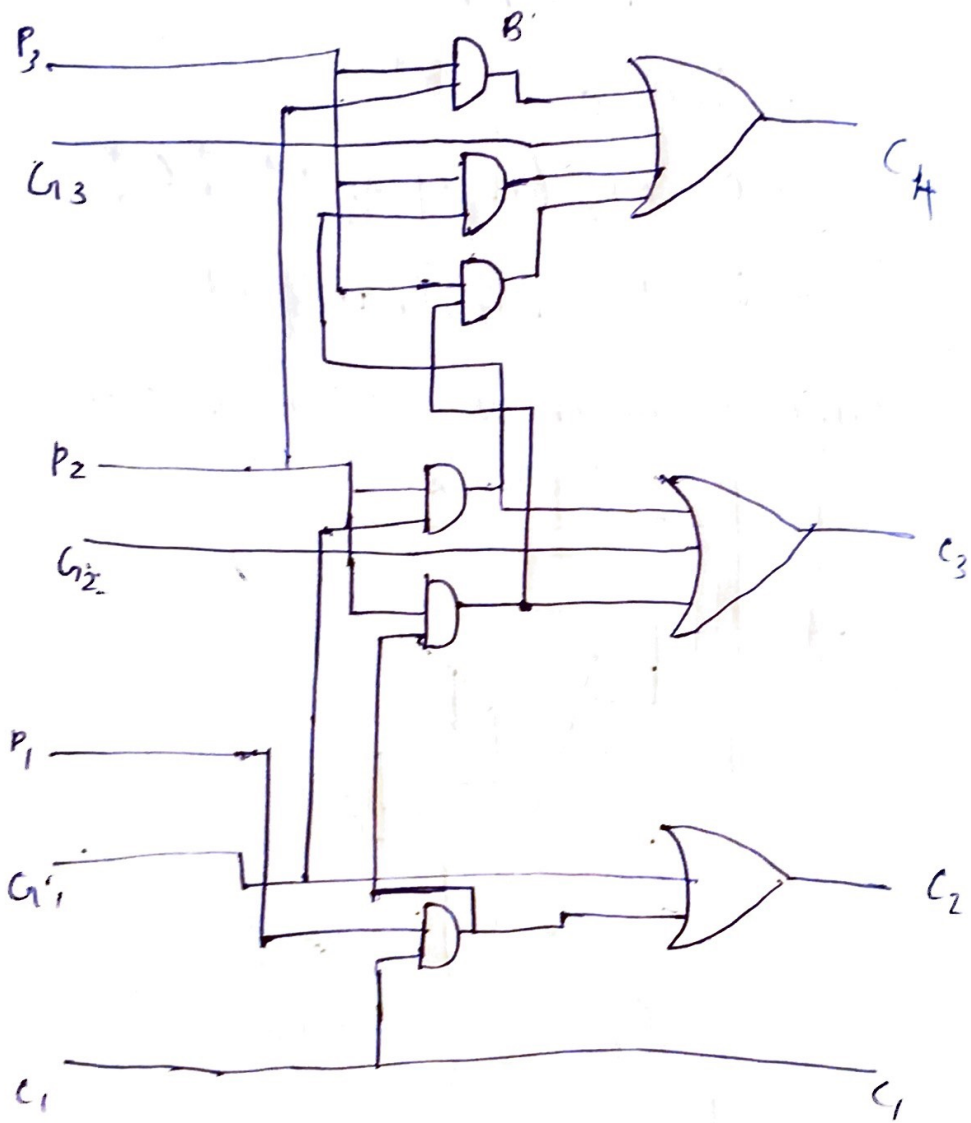
$$= G_3 + P_3 (G_2 + P_2 G_1 + P_2 P_1 C_1)$$

$$= G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 C_1$$

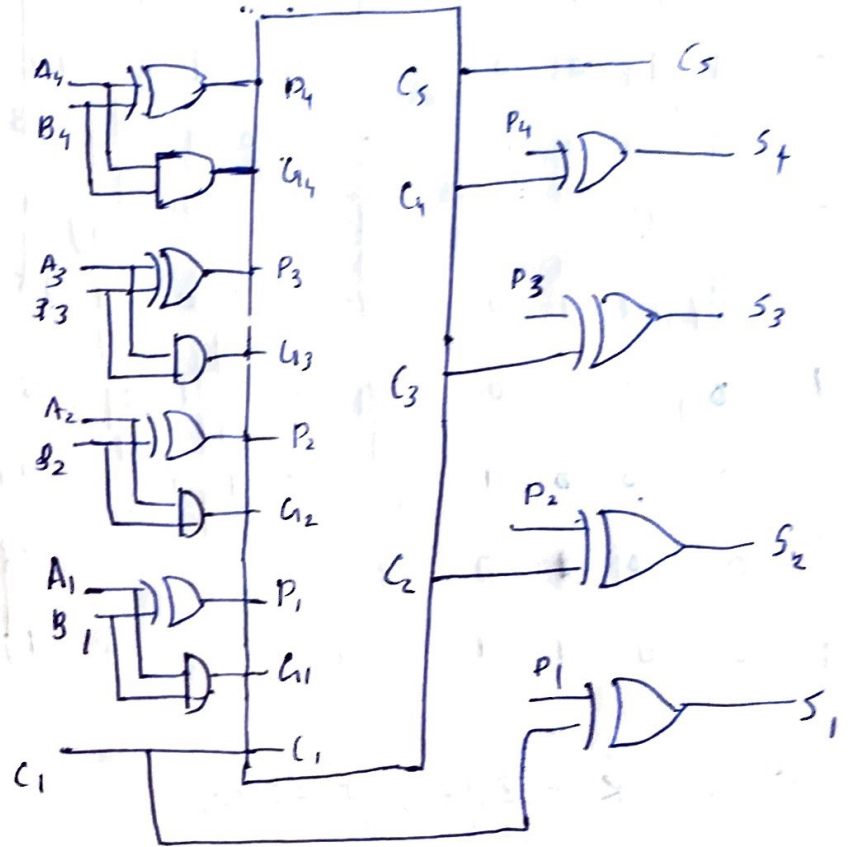
LOGIC DIAGRAM OF LOOK AHEAD CARRY

GENERATOR





4-bit binary parallel adder with look ahead carry generation

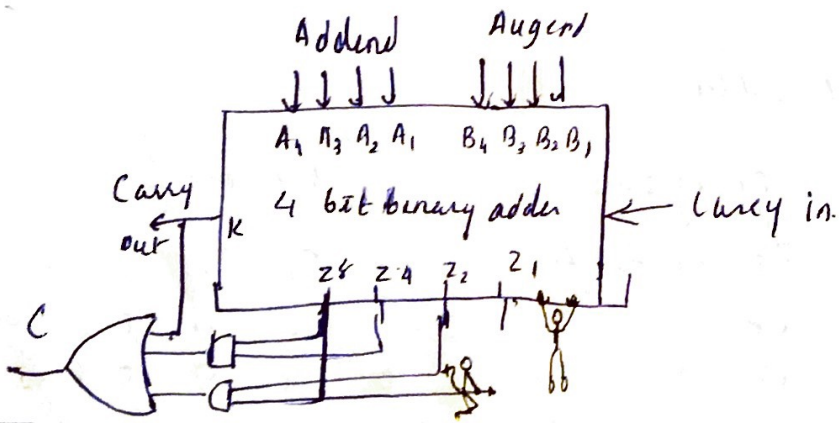


Decimal adder

K	Binary sum				C	BCD sum			
	Z ₈	Z ₄	Z ₂	Z ₁		S ₃	S ₄	S ₂	S ₁
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1

0	1	0	0	1	0	1	0	0	1	9
0	1	0	1	0	1	0	0	0	0	10
0	1	0	1	1	1	0	0	0	1	11
0	1	1	0	0	1	0	0	1	0	12
0	1	1	0	1	1	0	0	1	1	13
0	1	1	1	0	1	0	1	0	0	14
0	1	1	1	1	1	0	1	0	1	15
1	0	0	0	0	1	0	1	1	0	16
1	0	0	0	1	1	0	1	1	1	17
1	0	0	1	0	1	1	0	0	0	18
1	0	0	1	1	1	1	0	0	1	19

$$C = K + Z_5 Z_2 + Z_5 Z_4$$



MAGNITUDE COMPARATOR

- digital combinational circuit which compares the magnitudes of 2 binary numbers A and B and produces 3 o/p's $\rightarrow A > B, A = B, A < B$

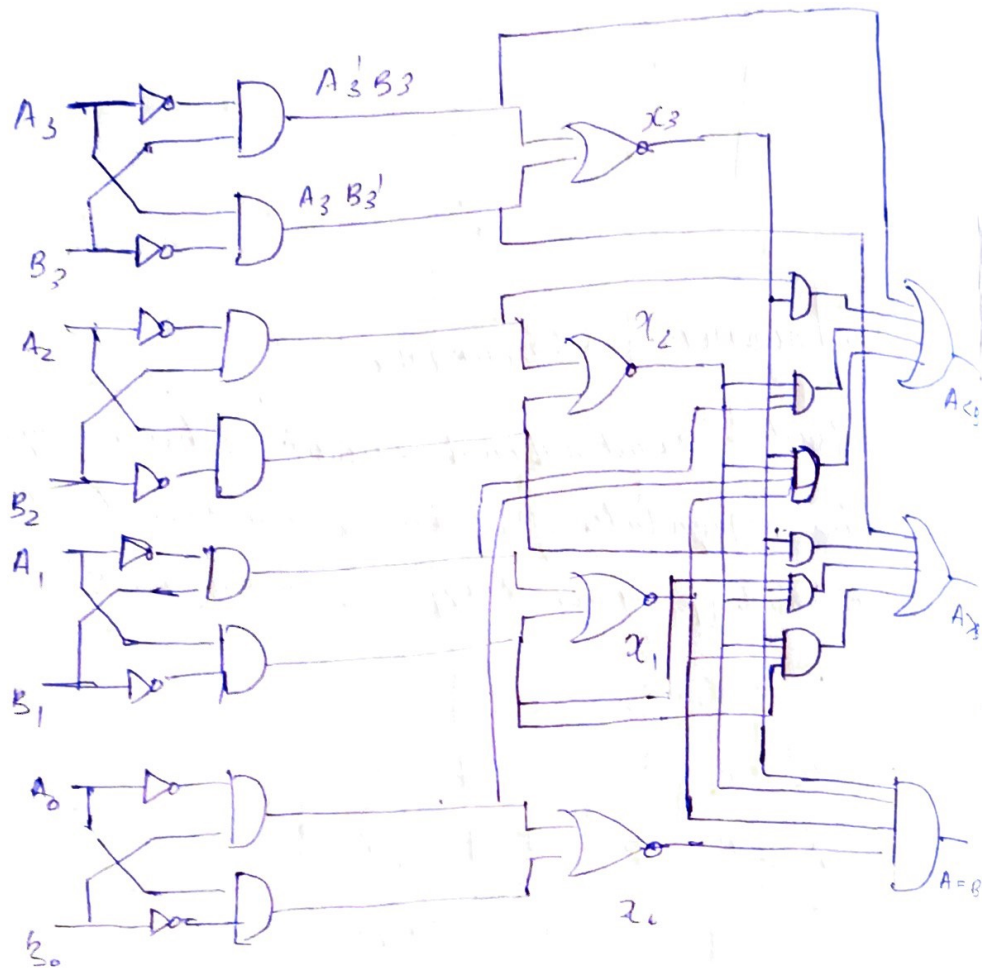
$$x_i = A_i B_i + A_i' B_i'$$

$$A = B \Rightarrow x_3 x_2 x_1 x_0$$

$$A > B \Rightarrow A_3 B_3' + x_3 A_2 B_2' + x_3 x_2 A_2 B_2' + x_3 x_2 x_1 A_1 B_1'$$

$$A < B \Rightarrow A_3' B_3 + x_3 A_2' B_2 + x_3 x_2 A_1' B_1 + x_3 x_2 x_1 A_0' B_0$$





Convert information from n lines to a maximum of 2^n outputs

A combinational circuit which converts input from n -lines to a maximum possible 2^n output lines. If the i/p has unused values then no. of o/p will be $< 2^n$
 n to m line decoder $m \leq 2^n$

eg. 3 to 8 decoder.

Design a BCD to decimal decoder (4 to 10 decoder)

BCD	o/p
Wxyz	$D_0 - D_9$

	yz	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$
$\bar{w}\bar{x}$	D_0	D_1	D_3	D_2
$\bar{w}x$	D_4	D_5	D_7	D_6
$w\bar{x}$	D_{10}	D_{11}	D_{13}	D_{12}
wx	D_8	D_9	D_{14}	D_{15}

$$D_0 = \bar{w}\bar{x}\bar{y}\bar{z}$$

$$D_1 = \bar{w}\bar{x}\bar{y}z$$

$$D_2 = \bar{w}\bar{x}y\bar{z}$$

$$D_3 = \bar{w}\bar{x}yz$$

$$D_4 = \bar{w}x\bar{y}\bar{z}$$

$$D_5 = \bar{w}x\bar{y}z$$

$$D_6 = \bar{w}xy\bar{z}$$

$$D_7 = \bar{w}xyz$$

$$D_8 = \bar{x}\bar{y}\bar{z} = w\bar{z}$$

$$D_9 = \bar{x}\bar{y}z = w\bar{z}$$

In circuit it will be converted first to binary

DECODER

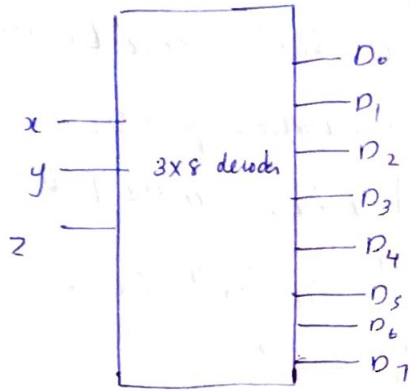
Convert 2×4 decoder \Rightarrow 2 i/p 4 o/p.

$$n \text{ i/p} \leq 2^n \text{ o/p}$$

If decimal decoder used if 4 i/p only 10 o/p

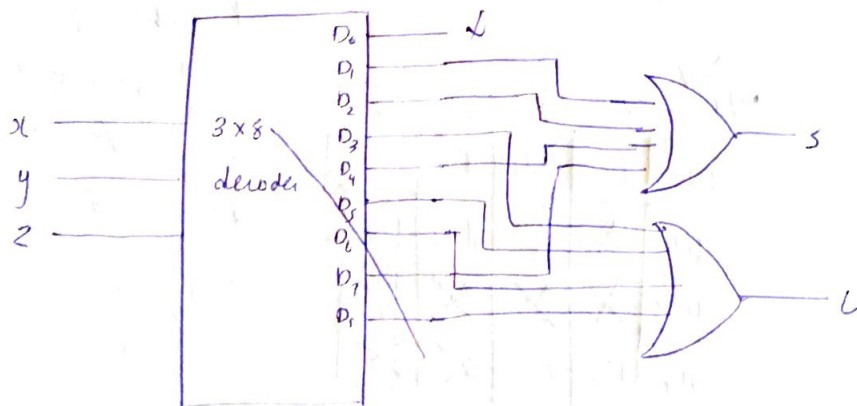
4 lines

3 to 8 decoder.



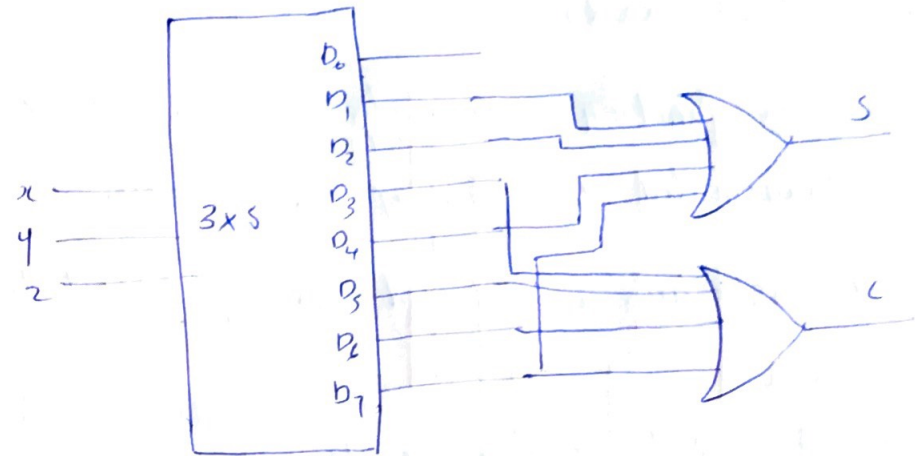
⇒ Design a full adder using a 3x8 decoder and 2 OR gates

COMBINATIONAL LOGIC IMPLEMENTATION



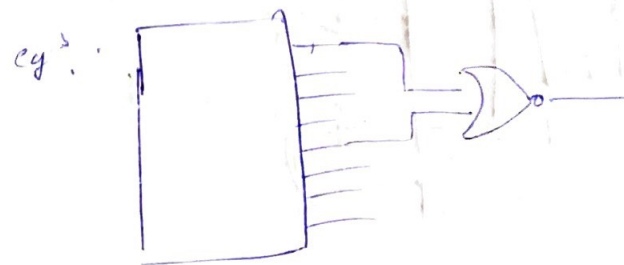
$$S = \Sigma(1, 2, 4, 7)$$

$$C = \Sigma(3, 5, 6, 7)$$



A decoder provides the 2^n min. terms of n i/p variable. Since any boolean function can be expressed in sum of minterm canonical form, we can use a decoder to generate the minterms and an external OR gate to form the sum.

ie Any combinational circuit with n i/p and m o/p can be implemented with a n -to- 2^n decoder and m OR gates.



$$S = \Sigma(1, 2, 3, 5, 6, 7)$$

0, 4

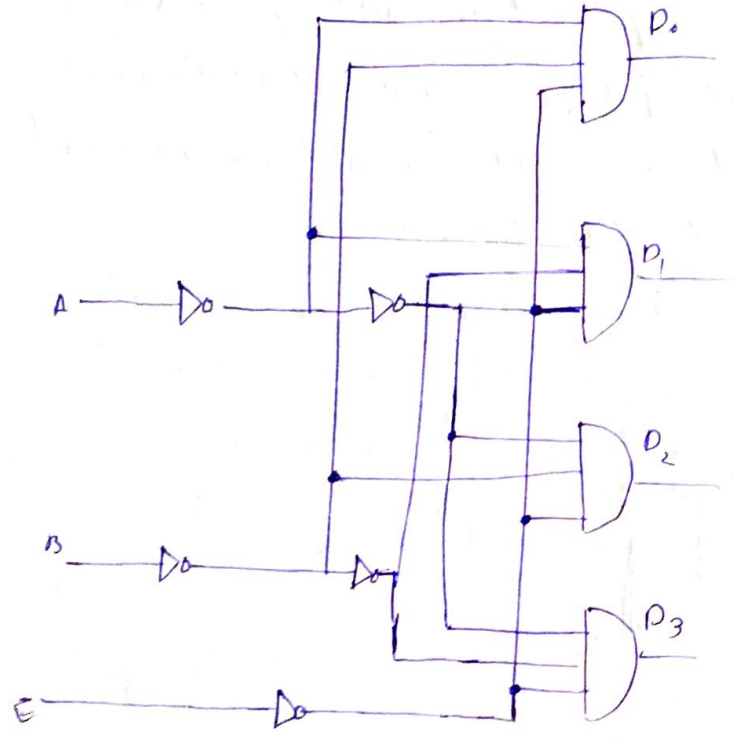
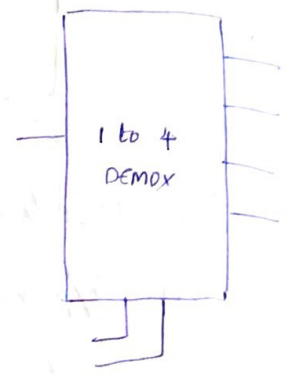
NOR gate

DEMULTIPLXERS:

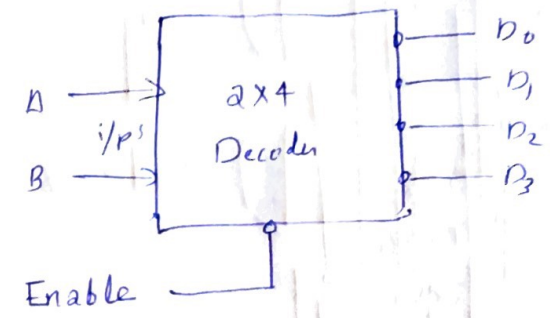
MUX \rightarrow Many \rightarrow One
 DEMUX \rightarrow One \rightarrow Many

Digital function where i/p from a single line is transmitted to one of the possible 2^n o/p lines. The o/p line is selected by 'n' select lines.

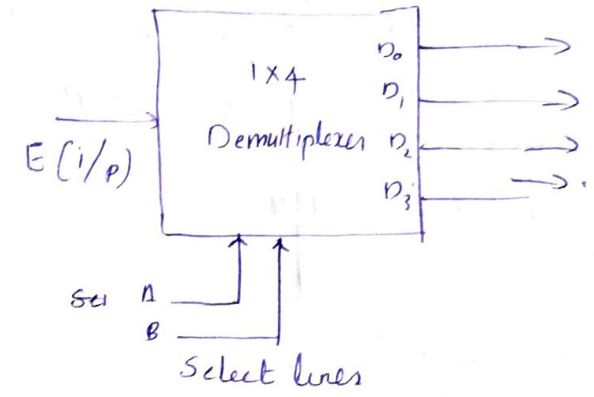
ex. A to 2 to 4 line decode with enable



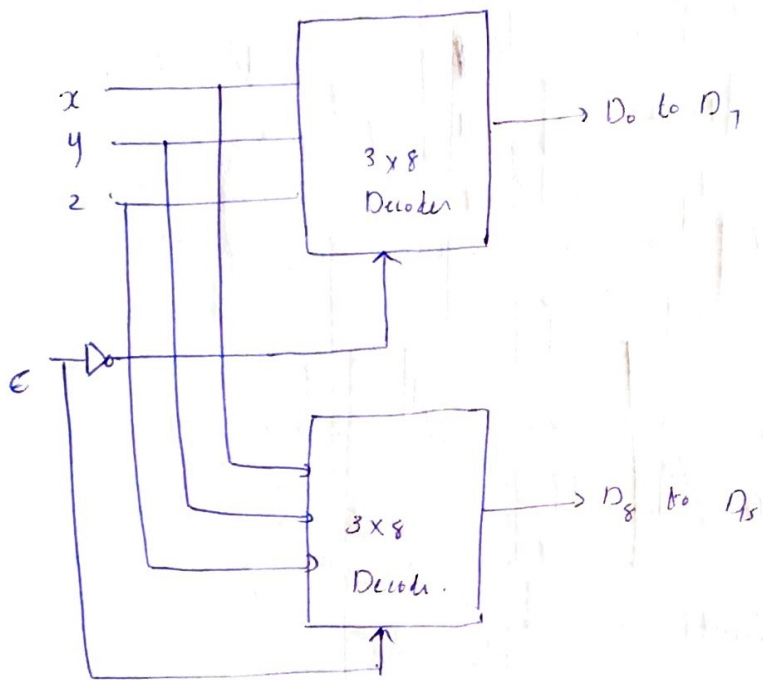
E	A	B	D_0	D_1	D_2	D_3
1	x	y	1	1	1	1
0	0	0	0	1	1	1
0	0	1	0	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0



Decoder with enable.



Design a 4x16 decoder with two 3x8 decoders



$2^4 = 16$

ENCODER - Reverse operation of a Decoder
 no. of i/p's $\rightarrow \leq 2^n$
 no. of o/p's $\rightarrow n$

ex: Octal to binary encoder.

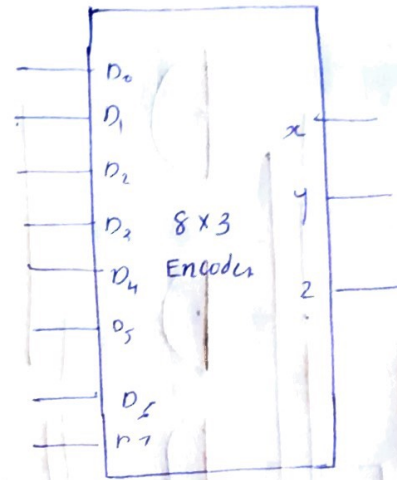
$D_0 - D_7$

$x = D_4 + D_5 + D_6 + D_7$

$y = D_2 + D_3 + D_6 + D_7$

$z = D_1 + D_3 + D_5 + D_7$

000	D_0
001	1
010	2
011	3
100	4
101	5
110	6
111	D_7



Encoder \rightarrow IC \rightarrow Priority encoder.

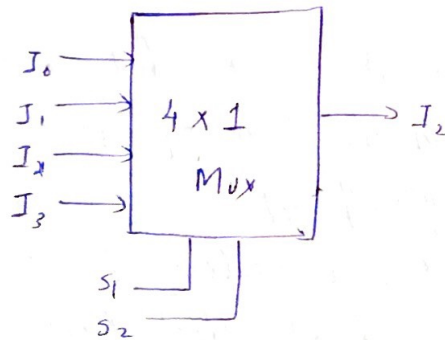
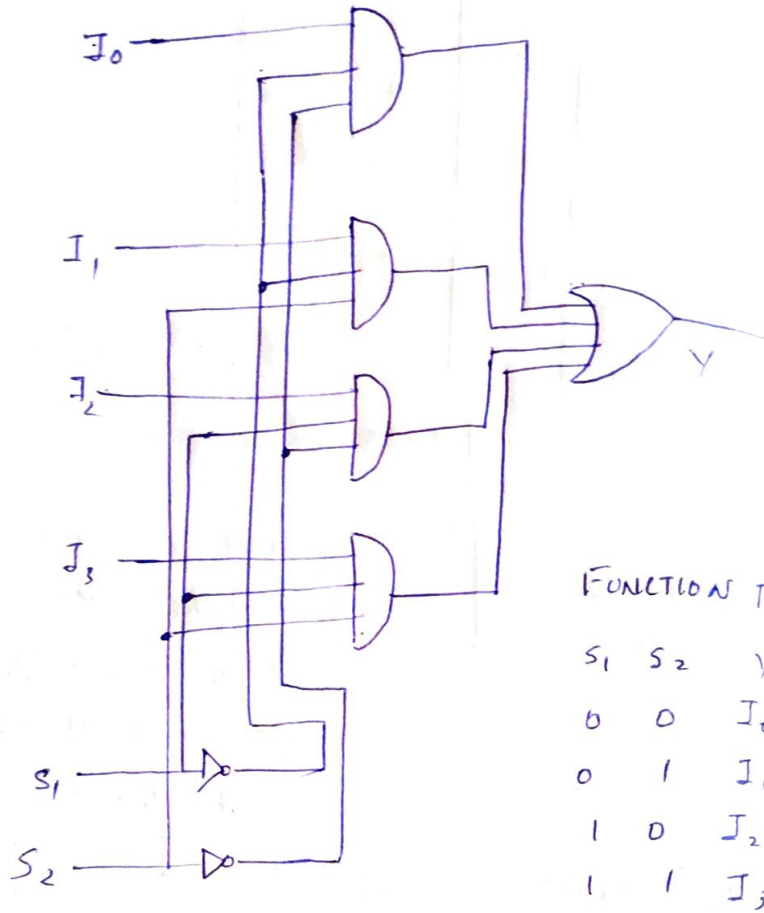
If 2 signals are simultaneously 1, then larger o/p is selected
 eg: If D_2 and $D_5 = 1$, we select the o/p 101.

MULTIPLEXER

\Rightarrow Data selection

- a combinational circuit that selects binary information from one of many i/p lines & directs it into a single o/p line.

The selection of a particular i/p line is controlled by a set of select lines



MUX \rightarrow with enable

\downarrow
Strobe (something that activates) multiplexer

BOOLEAN FUNCTION IMPLEMENTATION

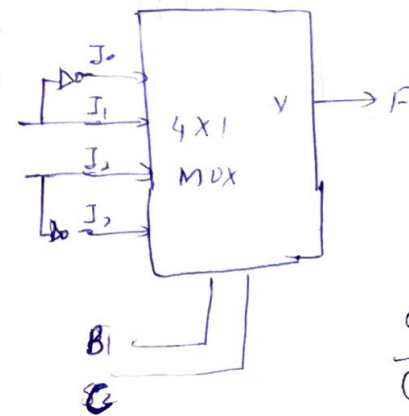
Any boolean function having n variables can be implemented in a 2^n to 1 MUX

eg. $F(A,B,C) = \Sigma(1,3,5,6)$

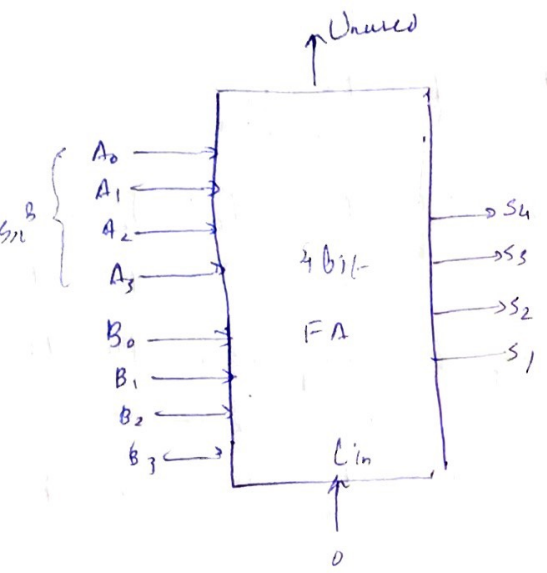
(A) IMPLEMENTATION TABLE

A	I_0	I_1	I_2	I_3
\bar{A}	0	1	2	3
A	4	5	6	7
	0	1	A	\bar{A}

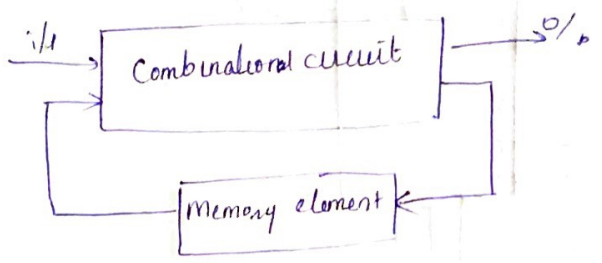
ABC	F
000	0
001	1
010	0
011	1
100	0
101	1
110	1
111	0



I_0	I_1	I_2	I_3
0	2	4	6
1	3	5	7
C	C	C	\bar{C}



SEQUENTIAL LOGIC

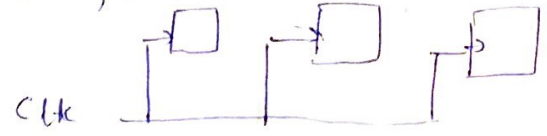


Synchronous ∴

→ Clock pulse



At fixed intervals



Asynchronous

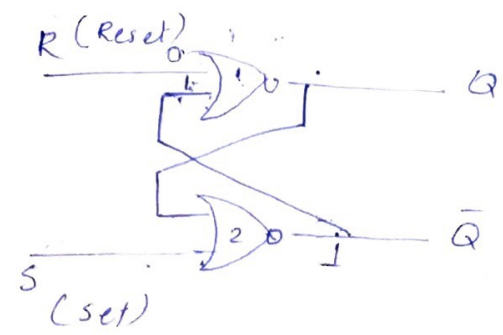


Each component triggered in a non-uniform manner

Memory element → Flip flop.

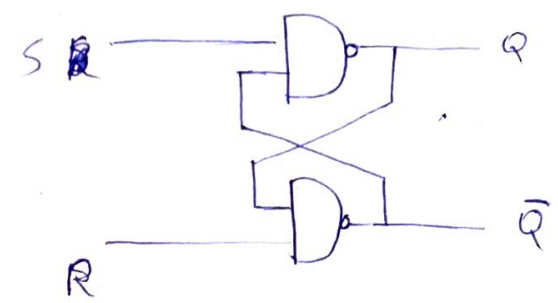
Basic flip flop.

NAND and NOR,



Reset = 0
Set = 1

S	R	Q	Q̄
1	0	1	0
0	0	1	0
0	1	0	1
0	0	0	1
1	1	0	0



S	R	Q	Q̄
1	0	0	1
0	0	0	1
0	1	1	0
0	0	1	0
0	0	1	0