LOGICS

- Study of reasoning

Preposition : Statement which is either true or false [lowercase]

Connectives: And ($\wedge$)

OR ($\vee$)

Not $\sim$ / $\bar{a}$

Implications $a \longrightarrow b$

biimplications $a \Longleftrightarrow b$

Proposition variables → denoted by lowercase letter

a;

Compound prepositions

- Connectives (AND, OR, NOT)

AND - Conjunction

OR - Disjunction

NOT - Negation

Consider the following

p: He is rich
q: He is generous
Write proposition which combines P and q using conjunction, disjunction and negation.

(∧) He is rich and generous ($P \wedge q$)

($p \vee q$) He is rich or generous

(~~P/q~~) ~~He is not~~ :

$\sim p$: He is not rich

$\sim q$: He is not generous

p: It is a hot day
q: The temp is 45°C

① $\sim p$: It is not a hot day

② $\sim(p \vee q)$: It is ~~neither~~ not a hot day ~~nor~~ and the temp is not 45°C

③ $\sim(p \wedge q)$: It is neither a hot day nor the temp is ~~is~~ not 45°C

④ $\sim(\sim p)$: It is a hot day

⑤ $p \vee q$: It is a hot day or the temp is 45°C

⑥ $\sim p \wedge \sim q$: It is not a hot day and the ~~the~~ temp is not 45°C

⑦ $\sim(\sim p \vee \sim q)$: It is a hot day and the temp is 45°C

Conditional Proposition

a) $p \rightarrow q$
(if p then q)
Hypothesis (Sufficient) → p; Conclusion (necessary) → q

p	q	($p \rightarrow q$)
0	0	T ①
0	1	T ①
1	0	F ⓪
1	1	T ①

b) $p \leftrightarrow q$ Biconditional Proposition
p iff q

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

VARIATIONS OF CONDITIONAL PROPOSITION

* Contrapositive : $(\sim q \rightarrow \sim p)$
* Converse : $q \rightarrow p$
* Inverse : $(\sim p \rightarrow \sim q)$

Q Construct truth table for following statements

1. $[(p \rightarrow q) \rightarrow [p \rightarrow \sim q)$
2. $(\sim q \rightarrow \sim p) \rightarrow (p \rightarrow q)$
3. $(p \rightarrow p) \vee (p \rightarrow \sim p)$
4. $p \Leftrightarrow (\sim p \vee \sim q)$

1 $p \rightarrow \sim q$ $[p \rightarrow q] \rightarrow [p \rightarrow \sim q]$

p	q	~q	p→~q	p→q	p→~q	(p→q)→p→~q
0	0	1	01	1	1	1
0	1	0	1	1	1	1
1	0	1	1	0	1	1
1	1	0	0	1	0	0

2

q	~q	p	~p	~q→~p
1	0	1	0	1
1	0	0	1	T
0	1	1	0	0
0	1	0	1	1

~~~q→~p → p→q~~

~q→~p	p→q	(~q→~p)→p→q
1	1	1
1	0	0
0	1	1
1	1	1

3

p	~p	p→p	p→~p
1	0	1	0
0	1	1	11

p→p	p→~p	p→p V p→~p
1	0	1
1	1	1

4

p	~p	q	~q	~p V ~q
0	1	0	1	1
0	1	1	0	1
1	0	0	1	1
1	0	1	0	0

p	~p V ~q	p⟺~pV~q
0	1	0
0	1	0
1	1	1
1	0	0

LOGICAL EQUIVALENCE

Some cases two compound proposition has same truth values no matter what truth values their constituent proposition has. Such proposition are said to be logically equivalent
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Suppose that the compound propositions ~~are~~ P, Q are made up of propositions $P_1, P_2 \ldots P_n$, we say that P, Q are logically equivalent and write it as $P \equiv Q$.

~~$\sim P \wedge$~~ $\sim(P \wedge Q) \equiv (\bar{P} \vee \bar{q})$

P	$\bar{P}$	Q	$\bar{Q}$	$P \wedge Q$	$P \vee Q$	$\sim(P \wedge Q)$	$\bar{P} \vee \bar{q}$
1	0	0	1	0	1	1	1
0	1	0	1	0	0	1	1
0	1	1	0	0	1	1	1
1	0	1	0	1	1	0	0

LOGICAL EQUIVALENCE

	Primal form	Dual form
Idempotent law	$p \vee p \equiv p$	$p \wedge p \equiv p$
Identity law	$p \vee F \equiv p$	$p \wedge T \equiv p$
Dominant law	$p \vee T \equiv T$	$p \wedge F \equiv F$
Complement law	$p \vee \bar{p} \equiv T$	$p \wedge \bar{p} \equiv F$
Commutative law	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Associative law	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive law	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Absorption law	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$

EQUIVALENCE INVOLVING CONDITIONALS

$p \rightarrow q \equiv \bar{p} \vee q$ $\qquad \bar{p} \rightarrow p$

$p \rightarrow q \equiv \bar{q} \rightarrow \bar{p}$

$p \vee q \equiv \bar{p} \rightarrow q$

$p \wedge q \equiv \sim(p \rightarrow \bar{q})$

$\sim(p \rightarrow q) \equiv p \wedge \bar{q}$

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Equivalence Involving Biconditionals

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\bar{p} \wedge \bar{q})$

$\sim(p \leftrightarrow q) \equiv p \leftrightarrow \bar{q}$

Construct the truth table

$(p \leftrightarrow q) \leftrightarrow ((p \wedge q)) \vee (\bar{p} \wedge \bar{q}))$

$(\bar{p} \vee \bar{q}) \leftrightarrow (p \leftrightarrow q)$

$(\bar{p} \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$

1.

p	$\bar{p}$	q	$\bar{q}$	$p \wedge q$	$\bar{p} \wedge \bar{q}$	$p \leftrightarrow q$	$(p \wedge q) \vee (\bar{p} \wedge \bar{q})$
0	1	0	1	0	1	1	1
0	1	1	0	0	0	0	0
1	0	0	1	0	0	0	0
1	0	1	0	1	0	1	1

$\therefore p \leftrightarrow q \Leftrightarrow ((p \wedge q) \vee (\bar{p} \wedge \bar{q}))$

2.

p	q	$\bar{p}$	$\bar{q}$	$p \leftrightarrow q$	$\bar{p} \vee \bar{q}$	$(\bar{p} \vee \bar{q}) \leftrightarrow (p \leftrightarrow q)$
0	0	1	1	1	1	1
0	1	1	0	0	1	0
1	0	0	1	0	1	0
1	1	0	0	1	0	0

3.

p	q	$\bar{p}$	$\bar{p} \vee q$	$\bar{p} \wedge q$	$p \wedge (p \wedge q)$	$((\bar{p} \wedge q) \wedge (p \wedge (p \wedge q)))$	$p \wedge$ …
0	0	1	0	0	0	0	0
0	1	1	1	1	0	0	0
1	0	0	1	0	0	0	0
1	1	0	1	0	1	1	1

$(\bar{p} \vee q) \wedge (p \wedge (p \wedge q))$

$= (\bar{p} \vee q) \wedge ((p \wedge p) \wedge q)$ — Associative Law

$= (\bar{p} \vee q) \wedge (p \wedge q)$

$= ((\bar{p} \vee q) \wedge q) \wedge p$

$= (q \wedge (q \vee \bar{p})) \wedge p$

$= q \wedge p$ — Absorbation law

$= p \wedge q$

Q. $p \to [q \to p] \equiv \bar{p} \to (p \to q)$

$p \to q$

LHS = ~~$\bar{p} \vee q \to p$~~

= ~~$\bar{p} \vee \bar{p} \to \bar{q}$~~

= ~~$(\bar{q} \to \bar{p}) \to \bar{p}$~~

$\bar{p} \to (\bar{p} \to \bar{q})$

$\bar{p} \vee (\bar{p} \to \bar{q})$

~~$\bar{p} \vee p \to \bar{q}$~~

RHS = $\bar{p} \to (p \to q)$ $\equiv \bar{p} \to (\bar{p} \vee q)$

= ~~$\bar{p} \vee (p \to \bar{q})$~~

$\therefore p \to [q \to p] \equiv \bar{p} \to [p \to q]$

LHS $\equiv \bar{p} \to [p \to q]$

$\equiv \neg p \vee [\neg q \vee p]$

$\equiv [\neg p \vee \neg q] \vee [\bar{p} \vee p]$

= $(\neg p \vee \neg q) \vee 1$

= 1

RHS $\equiv \bar{p} \to [\bar{p} \vee q]$

$\equiv p \vee [\bar{p} \vee q]$

= $(p \vee \bar{p}) \vee [p \vee q]$

= $[p \vee q] \vee 1$

= 1

1 → $\bar{p} \to (q \to r) \equiv q \to (p \vee r)$

~~2 → $p \to [q \to r] \equiv \bar{q} \vee r$~~

2 → $(p \vee q) \to r \equiv [p \to r] \wedge (q \wedge r)$

$P(x)$

QUANTIFIERS

$\forall x$

1. Universal quantifiers, for all, for every
2. Existential quantifiers, for some, there exist

$\exists x \, P(x)$

Let $P(x)$ be a stmnt involving variable x and let p be the set, we call P as a propos then fo wat P, if for each x in D $P(x)$ is a proposition, where D is domain of discourse of D.

2 in

1. $\bar{p} \to [q \to r] \equiv q \to [p \vee r]$

LHS: $\bar{p} \to [q \to r] \equiv \bar{\bar{p}} \to [\bar{q} \vee r]$

$\equiv \bar{\bar{p}} \vee [\bar{q} \vee r]$

$\equiv [\bar{\bar{p}} \vee \bar{q}] \vee [\bar{\bar{p}} \vee r]$

RHS: $q \to [p \vee r] \equiv \bar{q} \vee [p \vee r]$

$\equiv [\bar{q} \vee p] \vee [\bar{q} \vee r]$

1. $\bar{p} \to [q \to r] \equiv q \to [p \vee r]$

LHS $= \bar{p} \to [q \to r] \equiv \bar{p} \to [\bar{q} \vee r]$

$= p \vee [\bar{q} \vee r]$

$= p \vee \bar{q} \vee r$

RHS $= q \to [p \vee r] \equiv \bar{q} \vee p \vee r$

$= p \vee \bar{q} \vee r$ [Comm]

LHS = RHS

2. $[p \vee q] \to r \equiv [p \to r] \wedge [\bar{q} \wedge r]$

LHS $= [\overline{p \vee q}] \to r$

$[\overline{p \vee q}] \vee r = \bar{p} \wedge \bar{q} \vee r$

RHS $= [p \to r] \wedge [q \wedge r] = [\bar{p} \vee r] \wedge [r \wedge \bar{q}]$

[Comm]

$\leq \bar{p} \vee (r \wedge r) \wedge \bar{q} = \bar{p} \wedge \bar{q} \vee r =$ LHS

Q. Let $A(x)$: x has a white in color

$B(x)$: x is a polar bear

$C(x)$: x is found in cold region.

Translate following into simpler sentences.

1. $\exists x\, (B(x) \wedge \sim A(x))$
2. $\exists x\, (\sim C(x))$
3. $\forall x\, ((B(x) \wedge C(x)) \to A(x))$

1. There exists a polar bear whose color is not white
2. There exists animal which is not found in cold region

3. For all ~~animal which has white in color~~ Polar bear ~~and~~ found in polar region ~~has~~ implying that a polar it be white in color

Let $K(x)$: x is a two wheeler

$L(x)$: x is a scooter

$M(x)$: x is manufactured by bajaj

a) Every twowheeler is a scooter $\forall x [K(x) \rightarrow L(x)]$

b) There is a two wheeler that is not manufacture by bajaj $\exists x [K(x) \wedge (\sim M(x))]$

c) There is a 2 wheeler manufactured by bajaj that is not a scooter $\exists x [(K(x) \wedge M(x)) \wedge \sim L(x)]$

d) Every 2 wheeler that is a scooter is manufactured by bajaj

$\forall x [(K(x) \wedge L(x)) \wedge M(x)]$

When we negate a quantifier proposition, ie when a universal quantifier proposition is negated we get an existential quantifier proposition and vice versa.

→ ~~If~~ Negate following proposition

→ All boys can run faster than all girls

→ Some girls are more intelligent than all boys

→ Some students donot live in hostel

→ All students pass the semester exam

→ Some of the students are absent and the class room is empty

1. ~~All~~ ~~boys~~ ~~All~~ Some boys cannot run faster than ~~all~~ some girls

2. ~~Some~~ All girls are not ~~more~~ less intelligent than ~~all~~ some boys

3. ~~Some~~ All students live in hostels

4. ~~All~~ Some students didnot pass the semester exam

5. All students ~~areist~~ present absent and classroom is ~~not~~ not empty

Negate the proposition.

Q $\forall x\, P(x) \wedge \exists y\, Q y \Rightarrow \exists x \sim P(x) \vee \forall y \sim Q_y$

Q $\forall x\, P(x) \wedge \forall y\, Q y \Rightarrow \exists x \sim P(x) \vee \exists y \sim Q_y$

Q. $\exists x\, P(x) \vee \forall y\, Q(y) \Rightarrow \forall x \sim P(x) \wedge \exists y \sim Q_y$

Q $\exists x\, P(x) \vee \exists y\, Q y \Rightarrow \forall x \sim P(x) \wedge \forall y \sim Q_y$

Q $\forall x \forall y\, P(x,y) \Rightarrow \exists y \forall x\, P(x,y) \Rightarrow \forall x \exists y\, P(x,y)$

$\forall x \forall y\, P(x,y) \Rightarrow \exists x \forall y\, P(x,y) \Rightarrow \forall y \exists x\, P(x,y)$

NESTED QUANTITIES

$\forall x \forall y\, P(x,y) \Rightarrow \forall x (\forall y\, P(x,y)]$

$\Rightarrow \forall y [\forall x\, P(x,y)]$

$\exists x \exists y\, P(x,y) \Rightarrow \exists x [\exists y\, P(x,y)]$

$\Rightarrow \exists y [\exists x\, P(x,y)]$

The following simplification was true.

~~hold~~ be

ie ~~from this~~

ie, the negation of multiply quantified predicate formula, may be obtained by applying rules of negation from left to right

PROOF

Axioms

Definitions

Theorems — Lemmas / Corollaries

It is a proposition that has been proved to be true

Lemmas and corollaries are special kinds of theorems. ~~that is usually~~

Lemma is a theorem that is usually not too interesting in its own right but used for proving another theorem. Lem:

Corollaries is a theorem that follows

quickly from another theorem

Coro :

Proof by:

- Direct
- Indirect
- Contradiction
- Resolution
- Induction

Direct Proof:

To ~~sho~~ construct a proof of theorem - $\forall x (P(x) \to Q(x))$ D is the domain of this, So to direct proof we start by selecting an arbitrary chosen ~~number~~ mem ~~A~~ a of the domain D, then we show that $P(a) \to Q(a)$ is true.

Indirect Proof:

Consider implication $p \to q$, This implication is equivalent to $\sim q \to \sim p$

Inorder to show $p \to q$ is true, it can also show that implication $\sim q \to \sim p$ is true

Contradiction Method

It is established by assuming that hypothesis p/q is true and conclusion q is false and then using $\bar{p}$ and $\bar{q}$ as well as other axioms, definition and previously derived theorem derive a contradiction. The only diff btw assumption in direct and contradiction is the negated conclusion

RESOLUTION

If $p \vee q$ and $\bar{p} \vee r$ are true then $q \vee r$ is true. This statement can be verified by writing truth table bcoz resolution depends on single simple rule it is basis of many computer program to prove theorems. In proof by resolution

hypothesis and conclusion are written as clauses
[variables seperated by clauses

PROOF BY INDUCTION

Proof using mathematical induction has 2 parts.

1) they show that statement holds for positive Integer 1

2) It shows that statement holds for +ve integer then it must also holds for the next larger integer.

This mathematical induction is based on rule of inference. Tells us if P(1) and for all P(K) $P(K) \rightarrow P(K+1)$ are true for the domain of +ve integers, then $\forall n$ P(n) is true

The proof of mathematical statement by the bases of mathematical induction consists of 3 steps

1. Basic step

To show that P(n) is true for the particular non negative integer 0

2. Inductive hypothesis

To write inductive hypothesis, let k be an integer such that $k >= n$ and P(k) is true

3. Inductive step

To show that P(k+1) is true

Q. By using induction show that,

$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n[n+1][2n+1]}{6}$ $\forall$ integers $n >= 1$

1. Basic step

$P(1) = \frac{1[1+1][2\times1+1]}{6} = 1$

2. Inductive hypothesis. $[n = k]$

~~P[k] = k[k+1]?~~

$1^2 + 2^2 + 3^2 + \ldots + k^2 = \frac{k[k+1][2k+1]}{6}$

3. Inductive step

$1^2 + 2^2 + 3^2 + \ldots k^2 + (k+1)^2 = \frac{(k+1)[(k+1)+1][2k+2+1]}{6}$

$$\frac{k[k+1][2k+1]}{6} + [k+1]^2 = [k+1]\left[\frac{k[2k+1]}{6} + [k+1]\right]$$

$$= [k+1]\left[\frac{k[2k+1] + 6k+6}{6}\right]$$

$$= \frac{[k+1][2k^2+k+6k+6]}{6}$$

$$= \frac{[k+1][2k^2+4k+3k+6]}{6}$$

$$= \frac{[k+1]\,[2k+3]\;2k[k+2]\,3[k+2]}{6}$$

$$= \frac{[k+1][2k+3][k+2]}{6} = RHS.$$

3. ST. $1^3 + 2^3 + 3^3 + \dots n^3 = \left[\frac{n[n+1]}{2}\right]^2$

1. Base step

$P(1) = \left[\frac{1[1+1]}{2}\right]^2 = \left[\frac{2}{2}\right]^2 = 1$

2. Inductive hypothesis $n = k$

$1^3 + 2^3 + 3^3 \dots + k^3 = \left[\frac{k(k+1)}{2}\right]^2$

3. Inductive step

$1^3 + 2^3 + \dots + k^3 + [k+1]^3 = \left[\frac{[k+1][k+2]}{2}\right]^2$

$LHS = \left[\frac{k[k+1]}{2}\right]^2 + (k+1)^3 = (k+1)^2\left[\frac{k + [k^3+3k+k+1]2}{2}\right]$

$= \frac{[k+1]^2 [2k^3+8k+2+k]}{2}$

$= \frac{[k+1]^2 [2k^3+4k+2+5k]}{2}$

$= 2k[k$

$= [k+1]^2 \left[\frac{k}{2}\right]^2 + k+1$

$= [k+1]^2 \frac{k^2}{4} + k+1$

$= [k+1]^2 \cdot \frac{k^2+4k+4}{4} = \frac{[k+1]^2 (k+2)^2}{4}$

$= \left(\frac{(k+1)(k+2)}{2}\right)^2$

4. $1+4+7 \dots +3n-2 = \frac{n(3n-1)}{2}$

Sets, Relations & Functions

Finite Set

A set is said to be finite if its elements can be counted. eg: B = [x : x is a student of your school)

Infinite Set

A set is said to be infinite if it is not possible to count upto its last element

eg: A = [x : x is a natural number)

Disjoint Set

Two sets are said to be disjoint if they do not have any common element

eg: A = {1, 3, 5} B = {2, 4, 6} are disjoint set

Proper Subset

A proper subset of a set A is simply a set which contains some but not all of the objects in A.

eg: For the set, A = {1, 2, 3, 4, 5}, Q = {1, 3, 5} is a proper subset

Improper Subset

An improper subset is a subset which can be equal to the original set

eg: A = {1, 2, 3, 4}, subset, B = {1, 2, 3, 4} is an improper subset.

Null Set [Empty Set]

A set which has no element is said to be a null/empty/void set and is denoted by ϕ

eg: A = { x : x ∈ R and $x^2 + 1 = 0$ }

Power Set (P[A])

Power set of a set A is the set of all subsets of the given set

eg: A = {a, b} P[A] = { ϕ, {a}, {b}, {a, b} }

Universal Set

The set containing all objects or elements and of which all other sets are subsets.

eg: A = {1, 2} B = {3, 4, 5} C = {5, 6, 7}

U = {1, 2, 3, 4, 5, 6, 7}

Domain of a Relation

It is the set of all first elements of ordered pairs (x coordinates)

Range of a Relation

It is the set of all second elements of ordered pairs (y coordinates)

Complement of a Relation

Let R ⊆ S×T be a relation. The complement of this the relative complement of with respect to

$S \times T$: $S \times T(R) := \{(s,t) \in S \times T : (s,t) \notin R\}$

INVERSE OF A RELATION

An inverse relation is the set of ordered pairs obtained by interchanging the first and second elements of each pair in the original function

DOMAIN OF FUNCTION

The domain of a function f(x) is the set of all values for which the function is defined.

RANGE OF FUNCTION

The range of a function is the complete set of all possible resulting values of the dependent variable, after we have substituted the domain

CODOMAIN OF A FUNCTION

The codomain of a function is the set Y into which all of the output of the function is constrained to fall. It is the set Y in $f: X \to Y$

IMAGE

An image is the subset of a functions codomain which is the output of the function from a subset of its domain

TYPES OF FUNCTIONS

1. ONE TO ONE FUNCTION [INJECTIVE]
   Functions for which each element of the set A is mapped to a different element of the set B are said to be one to one.

2. ON TO FUNCTION [SURJECTIVE]
   A function f from a set X to a set Y is surjective, if for every element y in the codomain Y of f there is at least one element x in the domain X of f. sa

3. BIJECTIVE FUNCTION
   The function is bijective [one to one and onto] if each element of the codomain is mapped to by exactly one element of the domain.

4. MANY ONE FUNCTION
   A function can map more than one element of the set A to the same element of the set B. Such a type of function is called many function

5. MANYONE INTO FUNCTION
   Let $f: x \to Y$ the function f is called many one into function if and only if it both many one and into function.

6) Many ONE ON TO FUNCTION

Let $f: X \to Y$ the function f is called many one onto function if and only if it is both many one and onto

7) INTO FUNCTIONS

Let $f: X \to Y$ the function f is called an into function if the range of f is not equal to the co domain Y. ∴ there must be an element of co domain Y which is not the image of any element of domain X

8. ONE-ONE INTO FUNCTION

Let $f: X \to Y$, the function f is called one-one into function if diff elements of x have diff unique images of Y

EQUIVALENCE RELATION

① Reflexive $aRa \ \forall a \in A$

② Symmetric is $aRb \Rightarrow bRa$

③ Transitive · aRb & $bRc \Rightarrow aRc$

Let A be the set, $A = \{1, 2, 3, 4\}$, $R_1 = \{\}$.

$R_2 = \{(1,1), (2,2), (3,3)\}$ ~~(4,4)~~

$R_3 = \{(1,1), (2,2), (3,3), (2,1)\}$

$R_4 = \{(1,1), (1,3), (2,1), (3,1)\}$

$R_5 = \{(1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (3,1)\}$

$R_6 = \{(1,1)(1,2)(2,1)\}$

Check all whether are equivalent relation.

R_1, R_2 is an equivalent relation. (minm)

R_3 ✗

R_4 Not reflexive ✗

R_5 ✓

R_6

$R7 = A \times A$ maxm element to be a equivalent relation

EQUIVALENCE CLASS:

Q. Show that following relations are equivalent relations.

① R_1 = is a relation on set of integers such that aR_1b if and only iff $a = b$ or $a = -b$

② R_2 is a relation on the set of integers such that aR_2b iff $a \equiv b \pmod{m}$ where m is positive integer greater than 1.

③ R_3 is a relation on the set of real numbers such that aR_3b iff $(a-b)$ is an integer

Q. Determine whether the relation $S = \{(a,b) : a \geq b\}$ on the set R of Real numbers is an equivalence relation.

Q. Prove that the relation R (a−b) is divisible by 5 for all $a, b \in$ of I_+ is an equivalence relation

EQUIVALENCE CLASS:

A Consider an equivalence relation R on a set A, the equivalence class of an element A, $a \in A$, is the set of elements A to which a is related and it is denoted by [a]

$[x] = \{ y \mid y \in A \text{ and } (x,y) \in R \}$

$A = \{1,2,3,4,5\}$

$R = \{(1,1)(2,2)(3,3)(4,4)(5,5)(1,2)(2,1)(4,5)(5,4)\}$

$[1] = \{1,2\}$

$[2] = \{2,1\}$ or $(1,2)$

$[3] = \{3\}$

$[4] = \{4,5\}$

$[5] = \{4,5\}$

PARTITIONS

$\{1,2\}\{3\}\{4,5\}$

× × ×

$\{1,2\} \quad \{3\} \quad \{4,5\}$

$= ((1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,3)\ (4,4)\ (4,5)\ (5,4)\ (5,5))$

A partion {$A_1, A_2, \ldots A_i$} of a non empty set A is defined as collection of non empty subset of A such that (i) every element of A belongs to A_i ie Union of A_i = A

(ii) If A_i and A_j are distinct then $A_i \cap A_j$ equals null set ie partition divides element of set A into disjoint subsets

(iii) The subsets in a partition are called blocks or cells.

eg: Let R be equivalence relation on set A = {6,7,8,9,10} defined by R = {(6,6), (7,7), (8,8), (9,9), (10,10), (6,7) (7,6) (8,9) (9,8) (9,10) (10,9) (8,10)}

Find equivalent classes of R hence find partion of A corresponding to R.

[6] = {6, 7}

[7] = {7, 6}

[8] = {8, 9, 10}

[9] = {9, 8, 10}

[10] = {10, 9, 8}

Partition of A:

{6,7} {8,9,10}

× ×

{6,7} {8,9,10}

= (6,6) (6,7) (7,6), (7,7) (8,8) (8,9) (8,10) (9,8) (9,9) (9,10) (10,8) (10,9) (10,10)

③ Let A = {a,b,c,d,e} and

R = {(a,b) (a,a) (b,a) (b,b) (c,c) (d,d) (d,e) (e,d) (e,e)}

S = {(a,a) (b,b) (c,c) (d,d) (e,e) (a,c) (c,a) (d,e) (e,d)}

Be an ER on A. Determine the partitions corresponding to following (if it is an ER)

① R^{-1} ② $R \cup S$ ③ $R \cap S$

Let R is an ER on the set A = {p, q, r, s}
defined by partition P = {{p, s} {q, r}}
Determine the elements of ER & also find EC
of R

I

① R^{-1} = {(b,a) (a,a) (a,b) (b,b) (c,c) (d,d) (e,d) (d,e) (e,e)}

[b] = {b, a}   [d] = {d, e}

[a] = {a, b}   [e] = {e, d}

[c] = {c}

PARTITION = {a, b} {c} {d, e}

② R∪S = {(a,b) (a,a) (b,a) (b,b) (c,c) (d,d) (e,d) (d,e)
(e,e) (a,c) (c,a)}

[a] = {a, b, c}   [d] = {d, e}

[b] = {b, a}   [e] = {e, d}

[c] = {c, a}

PARTITION = {a, b, c} {d, e}

③ R∩S = {(a,a) (b,b) (c,c) (d,d) (d,d) (e,e)
(d,e) (e,d)}

[a] = {a}   [e] = {d, e}

[b] = {b}

[c] = {c}

[d] = {d, e}

PARTITION = {a} {b} {c} {d, e}

II   A = {p, q, r, s}

p = {p, s} {q, r}

×

{p, s} {q, r}

R = {(p,p) (p,s) (s,p) (s,s) (q,q) (q,r) (r,q)
(r,r)}

→ PARTIAL ORDER RELATION

- Reflexive
- Antisymmetric
- Transitive

Q. A = {1, 2, 3} R_1 = { } .   × Not reflexive

R_2 = {(1,1) (2,2) (3,3)} ✓

R_3 = {(1,1) (2,2) (3,3) (1,3) (2,3)} ✓

$R_4 = \{(1,1)(2,2)(3,3)(1,3)(2,3)\}$

$R_5 = \{(1,1)(1,2)(2,3)(1,3)\}$ ✗

$R_6 = \{(1,1)(1,3)(2,2)(2,3)(3,3)\}$ ✓

? Which of the following is not a partial order

$R_1 = \{(a,b) : (a,b) \in Z, a < b\}$ ✗

$R_2 = \{(a,b) \mid (a,b) \in Z, a \leq b\}$ ✓

$R_3 = \{(A,B) \mid A,B \in P(X), A \subseteq B\}$ ✓

$R_4 = \{(A,B) \mid a,b \in Z, b/a \in Z\}$ ✓

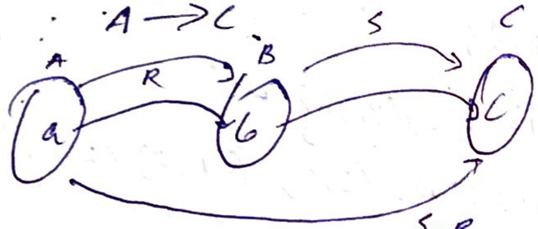
* Partial Order Relation
* Composition of relation
* Composition of function

COMPOSITION OF RELATION

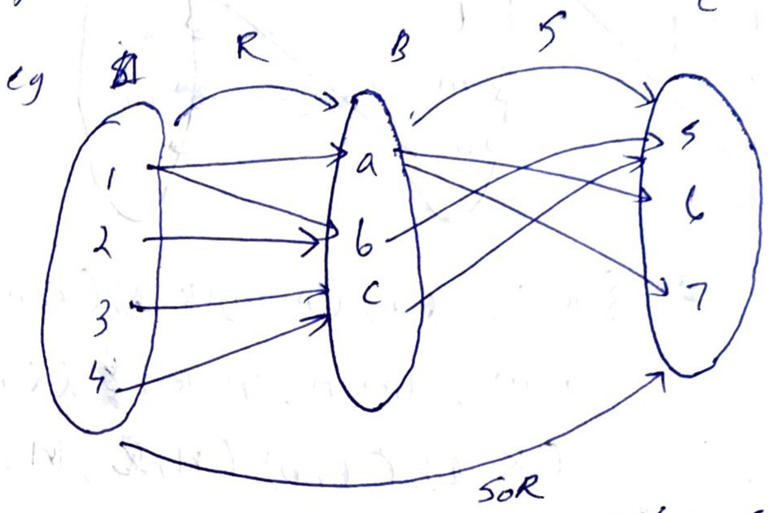
$R : A \to B$

$S : B \to C$

$S \circ R : A \to C$



If R is a relation from set A → B and set S is a relation B → C, then the composition of R and S denoted by $S \circ R$

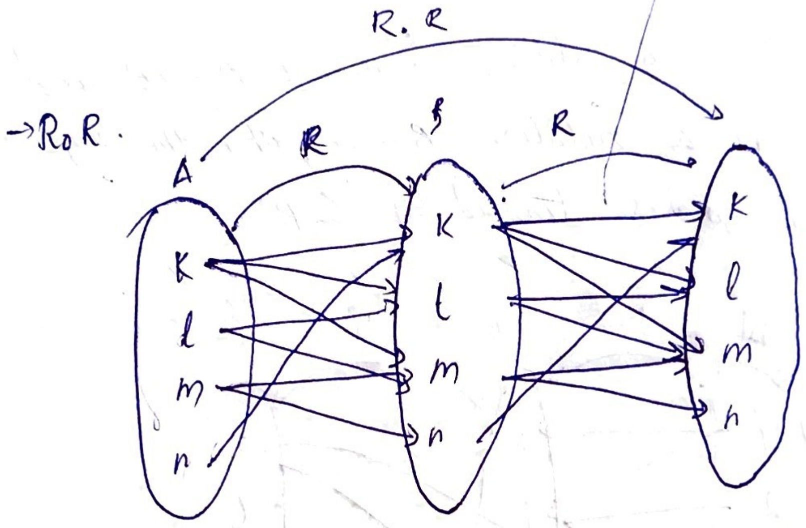


$S \circ R = \{(1,6)(1,7)(1,5)(2,5)(3,5)(4,5)\}$

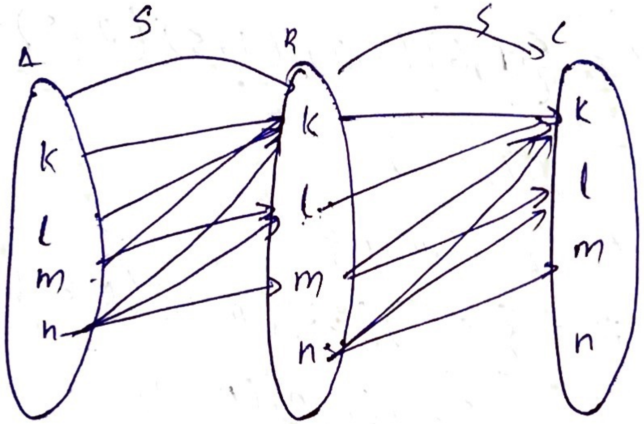
• Set $A = \{k, l, m, n\}$

$R = \{(k,k)(l,l)(m,m)(k,l)(k,m)(l,m)(m,n)(n,k)\}$

$S = \{(n,k)(n,l)(n,m)(m,k)(m,l)(l,k)(k,k)\}$

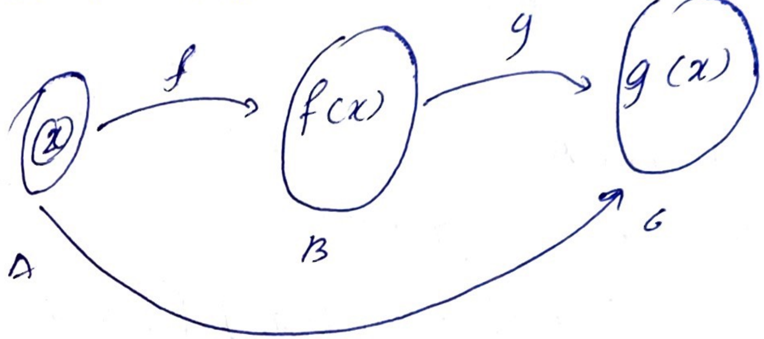


$R \circ R = \{(K,K)(l,l)(m,m)(K,l)(K,m)$
$(l,m)(m,n)(n,K)(K,n)$
$(K,l)(l,n)(n,l),(n,m)\}$



$S \circ S = \{(K,K)(M,K)(n,K)(n,l)$
$(l,K)\}$

COMPOSITION OF FUNCTION



$f: A \to B$

$g: B \to C$

$g \circ f(A): A \to C$

defined by $g \circ f(x) = g(f(x)) \; \forall x \in A$

→ Associative.

$h \circ (g \circ f) = (h \circ g) \circ f$

$f = A \to B$

$g: B \to C$

$h: C \to D$

→ $f: A \to B$

$g: B \to C$ are two functions then

$g \circ f: A \to C$ is an injection, surjection on bijection according as f and g are injective, surjections or bijections

Proof:

(i) Let $a_1, a_2 \in A$

Then $(g \circ f) a_1 = (g \circ f) a_2$

$\Rightarrow g[f(a_1)] = g[f(a_2)]$ ($\because$ g is injective)

$\Rightarrow a_1 = a_2$ ($\because$ f is injective)

$\therefore$ $g \circ f$ is injective

(ii) Let $c \in C$

Since g is onto, there is an element $b \in B$ such that $c = g(b)$

Since f is onto there is an element $a \in A$ such that $b = f(a)$

Now $g \circ f(a) = g[f(a)] = g(b) = c$

This means that $g \circ f : A \to C$ is onto

(iii) from i and ii it follows that $g \circ f$ is bijective when f and g are bijective

Q. If $S = \{1, 2, 3, 4, 5\}$ and if function $f, g, h : S \to S$ is given by

$f: \{(1,2)(2,1)(3,4)(4,5)(5,3)\}$

$f: \{(1,2)(2,1)(3,1)(4,2)(5,4)\}$

$g = \{(1,3)(2,5)(3,1)(4,3)(5,1)\}$

$h = \{(1,2)(2,2)(3,4)(4,3)(5,1)\}$

verify whether $f \circ g = g \circ f$

$f \circ g(1) = f(g(1)) = 4$

$f \circ g(2) = f(g(2)) = 3$

$f \circ g(3) = f(g(3)) = 1$

$f \circ g(4) = f(g(4)) = 1$

$f \circ g(5) = 5$

$f \circ g = \{(1,4)(2,3)(3,1)(4,1)(5,5)\}$

$g \circ f(1) = g(f(1)) = 5$

$g \circ f(2) = 3$

$g \circ f(3) = 2$

$g \circ f(4) = 4$

$g \circ f(5) = 1$

$g \circ f = \{(1,5)(2,3)(3,2)(4,4)(5,1)\}$

$f \circ g \neq g \circ f$

Consider f, g and h all functions from integer

$f(n) = n^2$ $g(n) = n+1$ $h(n) = n-1$

Determine $h \circ f \circ g$, $g \circ f \circ h$, $f \circ g \circ h$

2

$h \circ f \circ g(n) = h \circ f \circ (n+1)$

$= h \circ (n+1)^2$

$= (n+1)^2 - 1$

$g \circ f \circ h(n) = g \circ f \circ (n-1)$

$= g \circ (n-1)^2$

$= (n-1)^2 + 1$

$f \circ g \circ h(n) = n-1$

$f \circ g (n-1) = (n-1)+1$

$f \circ (n-1)+1 = n^2$

MODULE - II

ALGORITHM

- i/p - Algorithm receives i/ps 0 or more quantities
- ~~o/p~~ are externally supplied
- ~~Definiteness~~
- ~~Correctness~~
-

- o/p

From each set of input values the algorithm produces o/p values from a specified set. The o/p values are solution to problem

- Precision - Definiteness

Steps are precisely stated which instruction must be clear and unambiguous.

- Finiteness

Algorithm should produce desired o/p after a finite no. of steps for any i/p in the set

- Correctness

Algorithm should produce correct o/p

value for each set of i/p values.

EFFECTIVENESS

Every instruction must be very basic so that it can be carried out in principle and it in a finite amount of time

GENERALITY

Algorithm should be applicable for all problems not just for a particular set of i/p values.

Describe an algorithm for finding maxm value, of finite sequence of integers.

- Time / Space → Best case → Average case → Worst case

Asymptotic Notation.

→ Big Oh 'O',
→ Big Omega 'Ω'
→ Big Theta 'θ'

ω, o

Analysis of an algorithm refers to the process of deriving estimates for time and space needed to execute the algorithm

eg: If an algorithm takes n steps to solve a problem and another algorithm takes n^2 steps to solve same problem we prefer first algorithm. This estimation of time and space needed to execute the algorithm is called time and space complexity of the algorithm

There are 3 cases about the time complexity of an algorithm

a) best case time

It is minm no. of steps that can be executed for the given parameters

b) Worst case time

It is maxm no of steps that can be executed for the given parameters

c) Average case time

It is the avg no. of steps that can be executed for the given parameter

Asymptotic Notation

There are used to describe the execution time of an algorithm. There notation shows the order of growth of function. Here time taken by algorithm is mapped in terms of mathematical function.

Big - O Notation

The function $f(n) = O(g(n))$ iff there exists two +ve constants C and n_0 such that

$$f(n) \leq C * (g(n)) \quad \forall n, \; n \geq n_0$$

The bigh Oh notation provides an upper bound on the value of $f(n)$

Omega (Ω) Notation

The function $f(n) = \Omega(g(n))$ iff the exist two +ve constants C and n_0 such that

$$f(n) \geq C * \Omega(g(n))$$

The omega (Ω) notation provides the lower bound on the value of $f(n)$

Theta (θ) Notation

The function $f(n) = \theta(g(n))$ iff there exist three +ve constants C_1, C_2 and n_0 such that

$$C_1 * g(n) \leq f(n) \leq C_2 * g(n) \quad \forall n, \; n \geq n_0$$

The theta notation is more precise than both big-oh & omega notations. $f(n) = \theta(g(n))$ is both upper and lower bound on $f(n)$. Therefore g is an asymptote tight bound for f.